A Theory of Supply Function Choice and Aggregate Supply

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Abstract

Modern theories of aggregate supply are built on the foundation that firms set prices and commit to producing whatever the market demands. We remove this strategic restriction and allow firms to choose *supply functions*, mappings that describe the prices charged at each quantity of production. Theoretically, we characterize firms' optimal supply function choices in general equilibrium and study the resulting implications for aggregate supply. Aggregate supply flattens under lower inflation uncertainty, higher idiosyncratic demand uncertainty, and less elastic demand. Quantitatively, our theory can rationalize the flattening of aggregate supply during the Great Moderation and steepening during the 1970s and 2020s.

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1 Introduction

At the heart of modern macroeconomic models are monopolistic firms that make decisions under uncertainty. It is common to restrict these firms' decisions to a specific class: setting a price and committing to produce whatever the market demands. For example, price-setting is assumed in classic models of aggregate supply based on exogenous, infrequent adjustment (Taylor, 1980; Calvo, 1983), menu costs (Caplin and Spulber, 1987; Golosov and Lucas, 2007), and limited information (Mankiw and Reis, 2002; Woodford, 2003a). Price-setting firms are also at the core of the ubiquitous New Keynesian framework (Woodford, 2003b).

However, as has long been recognized (see *e.g.,* Grossman, 1981), price-setting is *not* typically an optimal way for a firm to behave and is, at some level, an *ad hoc* modeling assumption. Why should firms not be able to raise their prices when goods are flying off the shelves? In practice, they can and do: firms use policies like temporary sales and surge pricing to navigate changing demand conditions (Den Boer, 2015).

In this paper, we remove external restrictions on firms' pricing strategies and instead allow firms to choose any *supply function*: a mapping that describes the price charged at each quantity of production. Supply function choice is a standard approach in microeconomic theory to model firms' ability to adjust decisions to realized demand without imposing *ad hoc* strategic restrictions (*e.g.*, Grossman, 1981; Klemperer and Meyer, 1989). However, supply function choice has not previously been studied in general equilibrium, macroeconomic models. Our goal is to understand how this enriched model of pricing and production at the microeconomic level affects our understanding of the macroeconomy.

Introducing supply functions in an otherwise standard business-cycle model yields an aggregate supply curve with an endogenous slope. That is, the relative response of inflation and output to an aggregate demand shock depends on the interaction between uncertainty and market structure, precisely because these forces affect firms' optimal supply functions. Aggregate supply flattens, or aggregate demand shocks have bigger real and smaller nominal effects, under lower inflation uncertainty, higher idiosyncratic demand uncertainty, and less elastic demand. Quantitatively, the model's predictions for the slope of aggregate supply are consistent with time-series and cross-sectional evidence. Thus, supply functions provide a realistic and tractable foundation for a state-dependent aggregate supply curve.

Supply Function Choice of a Single Firm. We begin our analysis in partial equilibrium. We study a firm that faces a constant-price-elasticity demand curve and operates a constant-returns-to-scale production function. It has log-normal uncertainty about its competitors' prices, demand, productivity, input prices, and the stochastic discount factor.

Given its beliefs, the firm chooses a *supply function* $f : \mathbb{R}^2_{++} \to \mathbb{R}$ that defines the firm's supply curve as the locus of prices (p) and quantities (q) that solves $f(p,q) = 0$. Because the market clears, the firm produces and prices where the market demand curve intersects its supply curve. Internalizing this, the firm chooses its optimal, non-parametric supply function to maximize its expected real profits under the stochastic discount factor. The *ex ante* interpretation of the firm's choice is in line with the ECON 101 notion of a supply curve: a systematic plan relating the price that a firm charges and the quantity that it produces. We show that the model has a complementary interpretation in terms of the firm's information set for price and quantity choices: allowing firms to choose supply functions is isomorphic to allowing firms to condition prices and quantities on realized demand. This interpretation links our model theoretically to the notion of rational expectations equilibrium (Lucas, 1972) and practically to the aforementioned examples of firms adjusting prices in response to current demand conditions. We finally note that the model nests the polar cases of price- and quantity-setting studied in previous literature. By relaxing strategic restrictions, we allow firms to choose strategies that are potentially preferable to these extremes.

We solve for the optimal supply function and show that it is endogenously log-linear: $\log p = \alpha_0 + \alpha_1 \log q$. Thus, the firm's behavior in response to changes in market demand is described by its optimally chosen inverse supply elasticity α_1, the percentage by which the firm increases prices in response to a one percent increase in production. In turn, this elasticity depends on the firm's price elasticity of demand (*i.e.*, its market power) and its relative uncertainty about demand, competitors' prices, and real marginal costs. These relationships arise because uncertainty and market power shape firms' relative desires to hedge against different types of shocks.

Three comparative statics are particularly important for our macroeconomic analysis. First, higher uncertainty about firm-level demand pushes toward a lower α_1, or firms behaving more like price-setters. The limit case of price-setting perfectly insulates firms against demand shocks, as the optimal response of a firm to changing demand conditions is to set its relative price equal to a constant markup on its real marginal cost. Second, higher uncertainty about competitors' prices pushes toward a higher α_1, or firms' behaving more like quantity-setters. The limit case of quantity-setting perfectly insulates firms against shocks to competitors' prices as it allows the firm's relative price to adjust perfectly in response to such changes. Third, a lower elasticity of demand pushes toward a lower α_1, or firms behaving more like price-setters. More market power, thus defined, reduces the cost to the firm of setting the "wrong" price.

General Equilibrium: From Supply Functions to Aggregate Supply. We next embed supply-function choice in a business-cycle model with incomplete information (Woodford, 2003a; Hellwig and Venkateswaran, 2009). The model features: a representative household that demands differentiated consumption goods, demands money balances, and supplies labor; macroeconomic shocks to the money supply and productivity; microeconomic shocks to firm-specific wages, productivity, and demand; time-varying volatility for these shocks; and intermediate goods firms that choose supply functions in the face of endogenous uncertainty. Because of this uncertainty, shocks to the money supply can affect real aggregate output, as in Lucas (1972).

We first characterize aggregate outcomes given fixed firm-level supply functions. The unique log-linear equilibrium has an aggregate supply and aggregate demand representation. That is, real GDP and the price level can be determined as the intersection of two curves: an *aggregate supply* curve that is affected by productivity shocks but not monetary shocks and an *aggregate demand* curve that is affected by monetary shocks but not technology shocks. The "slope of aggregate supply" determines the relative within-period responses of the price level and real output to an aggregate demand shock.

The slope of aggregate supply depends critically on the slope of firms' microeconomic supply functions. Aggregate supply is inelastic—or, money is neutral—if and only if firms are quantity-setters. Aggregate supply is maximally elastic—or, money is as non-neutral as possible—if and only if firms are price-setters. These results are disquieting in light of the standard approach of *assuming* that firms set either prices or quantities. A key benefit of the supply function approach is that the analyst does not inadvertently impose restrictions on firms' supply function choices, but allows these choices to be made optimally. Between those extremes, the slope of aggregate supply is increasing in the slope of firm-level supply.

We next characterize how the slope of aggregate supply is endogenously determined via a fixed point relating macroeconomic uncertainty to firms' supply-function choice. This reveals feedback loops: uncertainty affects supply functions, which affects the slope of aggregate supply, and in turn shapes macroeconomic uncertainty. For a closed-form illustration, we first study a special case that balances strategic complementarity (from aggregate demand externalities) with substitutability (from wage pressure). In this case, the slope of aggregate supply decreases in a sufficient statistic for firms' uncertainty: their relative uncertainty about idiosyncratic demand shocks *vs.* aggregate demand shocks. Away from the special case of balanced strategic interaction, the model makes richer predictions in which the elasticity of demand and the volatility of productivity shocks also affect the slope of aggregate supply.

We finally observe that microeconomic and macroeconomic supply depend on relative rather than absolute uncertainties. Strikingly, even a vanishing amount of uncertainty can be consistent with *any* level of monetary non-neutrality, depending on the exact composition of uncertainty across different microeconomic and macroeconomic factors.

Our model predicts that the transmission of aggregate demand shocks is *state-dependent* because of firms' endogenous supply-function choices. This has several implications. First, more volatility in aggregate demand steepens aggregate supply. In our model, firms desire prices to respond more to demand in environments with more nominal uncertainty, and therefore endogenously lead demand shocks to have larger nominal effects and smaller real effects. Second, more volatility in *idiosyncratic* demand conditions has the opposite effect, flattening aggregate supply. Thus, counter-cyclical "risk shocks" at the microeconomic level (Bloom et al., 2018) increase the real effect of aggregate demand shocks. Third, a lower elasticity of demand (*i.e.*, more market power) flattens aggregate supply, due to forces missing *only* in the knife-edge case of exogenously assumed price-setting.

Quantification. To illustrate the plausibility and macroeconomic relevance of these predictions, we quantify their implications in two simple calculations. We first study the model's predictions across time in the United States. We calculate a time-varying slope of aggregate supply using a standard calibration for macroeconomic parameters plus estimates of time-varying volatility from a simple statistical model. Our quantification implies that the slope of aggregate supply is relatively flat in normal times but spikes dramatically during the inflationary episodes of the 1970s and 2020s, consistent with empirical findings (see, *e.g.*, Ball and Mazumder, 2011; Cerrato and Gitti, 2022). The key mechanism is that a spike in inflation uncertainty triggers firms to choose different supply functions, more aggressively varying prices with realized demand. In a second exercise, we study our model's implications for inflation-output tradeoffs across countries in the spirit of Lucas (1973). Our model predicts vastly heterogeneous slopes of aggregate supply that correlate positively with simple empirical proxies, but *not* with realized inflation or inflation volatility. Thus, our model's predictions based on relative uncertainties account for variation in the slope of aggregate supply beyond what is explained by models that tie the slope solely to the level of inflation and absolute inflation uncertainty.

Related Literature. Our methodological contribution is to derive aggregate supply in a business-cycle model from a foundation of supply function competition. Supply function competition has been extensively studied in microeconomic theory, industrial organization, and finance (*e.g.*, Grossman, 1981; Klemperer and Meyer, 1989; Vives, 2017). We contribute to this literature by analytically characterizing equilibrium supply functions with several new features: non-quadratic preferences; imperfect substitutability; multiple, correlated sources of uncertainty; and general equilibrium interactions in both input and product markets.

The closest analysis in the macroeconomics literature is performed by Reis (2006), who compares the extremes of price-setting and quantity-setting for a rationally inattentive firm in partial equilibrium. Our analysis goes beyond Reis' by studying completely flexible supply schedule choice, removing all *ad hoc* strategic restrictions on firms' choices, and studying general equilibrium.[1] The latter feature allows us to study the equilibrium relationship between supply function choice, macroeconomic dynamics, and the slope of aggregate supply.

Our finding that uncertainty shapes the slope of aggregate supply is shared with the classic "islands model" analysis of Lucas (1972). A shared methodological premise is that economic agents act on what they learn from endogenous objects. Our results for the slope of the aggregate supply curve differ substantially for two reasons. First, we study producers with market power, consistent with modern macroeconomic theory and evidence, instead of price-taking producers in competitive markets. Second, the inference problem that links uncertainty to supply decisions in our model arises for a different reason, without reference to the migration or physically separated markets. Rather, firms use the demand for their product as a noisy signal to infer their optimal price.

Our work is also distinguished from a literature that has pursued other avenues to reconcile Lucas' insights with non-competitive markets. Unlike Woodford (2003a), which restricts firms to price-setting, we allow firms to choose flexible schedules. This restores the spirit of Lucas' insight that firms can learn from market conditions in rational expectations equilibrium. Our analysis also suggests that existing conclusions about the link between information frictions and monetary non-neutrality are sensitive to strategic restrictions on firms: for example, if firms were restricted to set quantities in our model, money would be neutral despite information frictions. Hellwig and Venkateswaran (2009) share our premise of allowing firms to learn from demand conditions, but do not study the static fixed point that supply functions generate between firms' decisions and market information. This two-way feedback is at the core of our mechanism and our predictions.[2]

Outline. Section 2 solves for the firm's optimal supply function in partial equilibrium. Section 3 introduces a monetary business cycle model with supply functions. Section 4 characterizes equilibrium with supply function choice and shows how supply function choices affect aggregate supply. Section 5 quantifies the model's predictions. Section 6 concludes.

2 Supply Function Choice in Partial Equilibrium

In this section, we introduce our model of supply function choice for a single firm making decisions under uncertainty. We show that supply function choice is formally equivalent

[1] In Flynn et al. (2024), we study the problem of prices *vs.* quantities choice in general equilibrium and draw out implications for monetary policy.

[2] Lucas and Woodford (1993) and Eden (1994) study markets with *ex ante* capacity investment and sequential transactions as a way to model learning from demand conditions. These authors also do not study the static fixed-point between uncertainty and market information.

to allowing firms to learn from their demand and update their pricing strategies accordingly. Our main result in this section shows that optimal supply functions are log-linear and characterizes their slope in terms of firms' uncertainty and the elasticity of demand.

2.1 The Firm's Problem

Environment. A firm produces output $q \in \mathbb{R}_+$ via a constant-returns-to-scale production technology using a single input $x \in \mathbb{R}_+$:

$$q = \Theta x \tag{1}$$

where $\Theta \in \mathbb{R}_{++}$ is the firm's Hicks-neutral productivity. The firm can purchase the input at price $p_x \in \mathbb{R}_{++}$. The firm faces a constant-elasticity-of-demand demand curve given by:

$$\frac{p}{P} = \left(\frac{q}{\Psi}\right)^{-\frac{1}{\eta}} \tag{2}$$

where $p \in \mathbb{R}_+$ is the market price, $\Psi \in \mathbb{R}_{++}$ is a demand shifter, $P \in \mathbb{R}_{++}$ is the aggregate price level, and $\eta > 1$ is the price elasticity of demand. We interpret the elasticity of demand as an (inverse) measure of market power: when η is high, the quantity demanded is more sensitive to the price. The firm's profits are priced according to a real stochastic discount factor $\Lambda \in \mathbb{R}_{++}$. For simplicity, we define the firm's real marginal cost as $\mathcal{M} = P^{-1}\Theta^{-1}p_x$.

At the beginning of the decision period, the firm is uncertain about demand, costs, others' prices, and the stochastic discount factor (SDF). Specifically, they believe that the state $(\Psi, \mathcal{M}, P, \Lambda)$ follows a log-normal distribution with mean μ and variance Σ. The firm's payoff is given by its expected real profits (revenue minus costs), as priced by the real SDF:

$$\mathbb{E}\left[\Lambda\left(\frac{p}{P} - \mathcal{M}\right)q\right] \tag{3}$$

where $\mathbb{E}[\cdot]$ is the firm's expectation given some joint beliefs about $(\Lambda, P, \mathcal{M}, \Psi, p, q)$.

Supply-Function Choice. The firm implements price-quantity pairs described by the implicit equation $f(p, q) = 0$ where $f : \mathbb{R}^2_{++} \to \mathbb{R}$. We will refer to f as the supply function. Price-setting is nested as a case in which $f(p, q) \equiv f^P(p)$. Quantity-setting is nested as a case in which $f(p, q) \equiv f^Q(q)$. More generally, we allow plans to be given by any non-parametric function f, allowing for possible non-monotonicity and discontinuities.

After choosing a supply function f, and following the realization of Ψ and P, the firm produces at a point where f intersects the demand curve. That is, the market clears. To formalize this, we define the *nominal demand state* $z = \Psi P^\eta$ and rewrite the demand curve as $q = zp^{-\eta}$. Thus, having set f and following the realization of z, the firm's price is given by some solution $\hat{p}$ to the equation $f(\hat{p}, z\hat{p}^{-\eta}) = 0$ with the realized quantity being $\hat{q} = z\hat{p}^{-\eta}$. We assume that the firm chooses the profit-maximizing selection from the set of solutions if there are many and does not produce if there is no solution. Given a supply function f, we let $H(f)$ be the induced joint distribution over $(\Lambda, P, \mathcal{M}, \Psi, p, q)$ given the firm's prior beliefs. The firm's problem of choosing an optimal supply function is therefore equivalently stated as either of the following maximization problems:

$$\sup_{f:\mathbb{R}^2_{++}\to\mathbb{R}} \mathbb{E}_{H(f)} \left[\Lambda \left(\frac{p}{P} - \mathcal{M}\right) q\right] \iff \sup_{\hat{p}(z)} \mathbb{E}\left[\Lambda \left(\frac{\hat{p}(z)}{P} - \mathcal{M}\right) z\hat{p}(z)^{-\eta} \middle| z\right] \text{ for all } z \in \mathbb{R}_+ \quad (4)$$

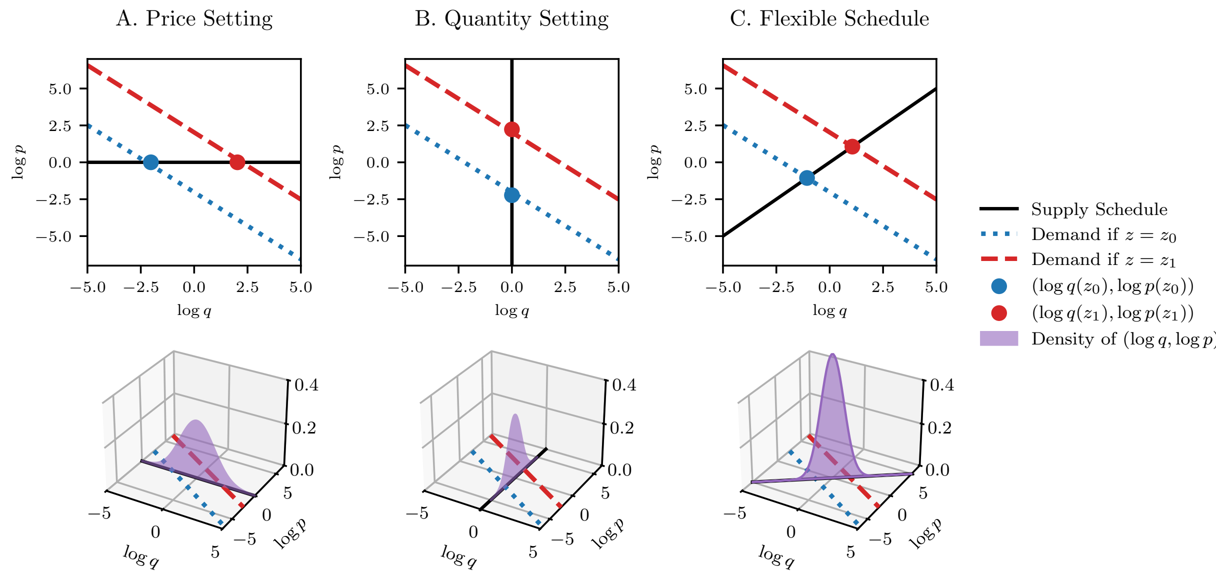
While mathematically equivalent, these two formulations of the problem provide two different economic intuitions for how the firm behaves. Under the first formulation, the interpretation is that the firm chooses its supply curve *ex ante*, knowing that it will price and produce where its supply curve meets the demand curve. This is like the ECON 101 notion of a supply curve as the firm's plan linking production and prices. Under the second formulation, the interpretation is that the firm prices in the *interim*: it is *as if* the firm sees the state of its demand, updates its beliefs, and then sets its optimal price. That is, supply functions allow the firm to condition its price and quantity on the strength of demand.

In Figure 1, we illustrate how different supply functions translate into price-quantity outcomes. The first row illustrates market clearing by showing different supply function intersected with two demand curves, corresponding to "high" and "low" realizations. The second row illustrates the induced joint distribution of prices and quantities. Panel A shows a "price-setting" supply function, $f(p, q) = 1-p$. In this case, a firm responds to low demand by producing less and responds to high demand by producing more in order to maintain a fixed price. The "quantity-setting" policy (panel B), $f(p, q) = 1 - q$, does the opposite: in this case, the firm aggressively decreases the price of low-demand goods and increases the price of high-demand goods to fix the quantity sold. The supply function in panel C, $f(p, q) = 1 - \frac{p}{q}$, allows both prices and quantities to increase with demand. This describes a firm with less extreme dynamic pricing: high-demand states have higher prices and volumes, and low-demand states have lower prices and volumes. In our model, the firm picks the optimal supply function given its uncertainty about economic conditions.

Interpreting Supply Functions. We argue that there are strong theoretical and empirical grounds to study supply function choice as a benchmark model of firm choice under uncertainty.

First, the supply function model is consistent with firms' using all valuable information

Figure 1: An Illustration of Supply-Function Choice



Notes: The columns correspond to different supply functions. The top row illustrates *ex post* market clearing for two realizations of the demand curve. The bottom curve illustrates the induced joint distribution of quantities and prices given log-normal uncertainty about z.

revealed by market clearing. This premise is central to the modern paradigm of rational expectations equilibrium (REE, Lucas, 1972). It is of course possible that firms are hindered in incorporating this information by *ex post* costs to adjust quantities and prices, for example due to technological constraints or nominal rigidities. But, as we further clarify in Section 2.3, any frictions that induce *finite* adjustment costs are conceptually straightforward to incorporate into the supply-function model—whereas only *infinite* adjustment costs on specific margins can justify the standard models of price-setting and quantity-setting. From this perspective, the only tenable arguments against allowing for supply functions are that firms do not learn in the manner required by REE or that adjustments are infinitely costly.

Second, supply function choice does not require that firms commit to plans that are revealed to be suboptimal by demand conditions. For example, in the typical model of price-setting, a firm could lose money (to an arbitrary extent) to honor its fixed-price commitment in a state of the world in which goods are “flying off the shelves.” Our supply function model can be thought of as formalizing the scenario in which a firm can reconsider its plans after every realization of demand.

Third, the supply function model decouples “price inertia” from “price stickiness,” but is not incompatible with the latter. As in other models of aggregate supply based on incomplete information (*e.g.*, Lucas, 1972; Mankiw and Reis, 2002; Woodford, 2003a), our model predicts that the prices charged by an individual firm change in every period. We adopt this intentionally extreme view to highlight how the novel mechanisms of our model can translate "flexible" prices into sluggish responses of the aggregate price level to shocks (see Section 4). Nonetheless, it is conceptually straightforward to extend the model to allow for microeconomic price rigidity (see Section 2.3). In this case, the supply functions model still has appealing theoretical properties relative to the benchmarks of price- and quantity-setting—no firm regrets its price after observing market clearing—as well as materially different predictions, upon which we later elaborate and test.

Fourth, as a practical matter, there are many pricing strategies that could be well described as supply functions that incorporate information from contemporaneous demand conditions into prices. This is common for many goods and services that feature "dynamic pricing," for example in markets for electricity, gasoline, e-commerce, ride-sharing, and entertainment (see *e.g.,* Den Boer, 2015). Klemperer and Meyer (1989) provide two concrete examples of firms that *de facto* implement supply schedules: management consultants who do not post prices, but instead vary them as a function of the quantity of services provided, and airlines that use computer software to put seats on discount depending on how many are currently sold. Crucially, in these examples, firms' choices do not seem completely constrained by technological necessity: they could in principle vary prices more or less aggressively depending on what is more profitable. Finally, varying prices with demand is not a new innovation: negotiated rather than posted prices were the norm throughout human history until the invention of the price tag in the mid-19th century (Phillips, 2012).

2.2 The Optimal Supply Function

We now study the optimal supply function. The following result characterizes the firm's optimal policy in closed form and allows us to illustrate comparative statics in the extent of uncertainty and the price elasticity of demand.

Theorem 1 (The Optimal Supply Function)**.** *Any optimal supply function is almost everywhere given by:*

$$f(p,q) = \log p - \alpha_0 - \alpha_1 \log q \tag{5}$$

where the slope of the optimal price-quantity locus, $\alpha_1 \in \overline{\mathbb{R}}$, is given by:

$$\alpha_1 = \frac{\eta\sigma_P^2 + \sigma_{\mathcal{M},\Psi} + \sigma_{P,\Psi} + \eta\sigma_{\mathcal{M},P}}{\sigma_\Psi^2 - \eta\sigma_{\mathcal{M},\Psi} + \eta\sigma_{P,\Psi} - \eta^2\sigma_{\mathcal{M},P}} \tag{6}$$

Proof. See Appendix A.1. □

Understanding the Result. To provide intuition for this result, it is helpful to first sketch its proof. As observed above (Equation 4), the problem of choosing an optimal supply function *ex ante* can be recast as a problem of choosing price-quantity pairs $(p(z), q(z))$ that are indexed by the realization of the nominal demand state $z = \Psi P^\eta$ and are such that the market clears: $(p(z), q(z)) = (p(z), zp(z)^{-\eta})$. Intuitively, when setting a supply schedule, the firm anticipates that it will produce where the demand curve intersects the supply function. Thus, as the demand curve is indexed by z, it is as if the firm chooses a z-contingent price-quantity plan. Importantly, it does so *without additional constraints*. We emphasize that this differs from the standard case of price- and quantity-setting due to the *lack of constraints* and not the contingency of choices on demand. Under price-setting, firms implement z-contingent price-quantity pairs of the constrained form $(p(z), q(z)) = (\bar{p}, z\bar{p}^{-\eta})$, where $\bar{p}$ is a fixed price and $z\bar{p}^{-\eta}$ is the quantity that clears the market; and similarly, under quantity-setting, firms implement z-contingent price-quantity pairs of the constrained form $(p(z), q(z)) = (z^{1/\eta}\bar{q}, \bar{q})$.

When choosing flexible supply schedules, firms can therefore freely incorporate information from the nominal demand state z into their optimal choices. To see this, we note that a necessary condition for optimality is that, for any given realization $z = t$, there is no local benefit to changing the price $p(t)$. Taking a first-order condition at each $z = t$, we find that the firm equates the marginal revenue and cost effects of raising the price:

$$\underbrace{\mathbb{E}\left[(\eta - 1)z\Lambda P^{-1}p(z)^{-\eta} \mid z = t\right]}_{\text{Expected revenue effect}} = \underbrace{\mathbb{E}\left[\eta z\Lambda\mathcal{M}p(z)^{-\eta-1} \mid z = t\right]}_{\text{Expected cost effect}} \tag{7}$$

This principle of equating expected benefits and costs would apply in a broader class of models with different production technologies and demand curves (see Section 2.3), although the specific expressions for these terms in Equation 7 rely on our exact assumptions. Rearranging terms, we observe that, for almost all $t \in \mathbb{R}_{++}$, the optimal price must satisfy:

$$p(t) = \frac{\eta}{\eta - 1}\frac{\mathbb{E}[\Lambda\mathcal{M} \mid z = t]}{\mathbb{E}[\Lambda P^{-1} \mid z = t]} \quad \text{and} \quad q(t) = tp(t)^{-\eta} \tag{8}$$

This resembles the standard optimality condition for monopolistic price-setting ("markup over marginal cost"), with the key difference that it conditions on nominal demand z. Outcomes under optimal rules therefore differ from outcomes under optimal price-setting due to the firm's ability to make inferences about the stochastic discount factor, real marginal costs, and the price level. Finally, we exploit the joint log-normality of the variables in the firm's problem to solve Equation 8 for an exact log-linear relationship between prices and quantities. Without log-normality, Equation 8 could be solved using numerical techniques.

It remains to explain why the optimal inverse supply elasticity takes the form given in

Equation 6. This specific form arises because α_1 is the relative rate at which the firm wants log prices and log quantities to increase with the nominal demand state $\log z$:

$$\alpha_1 = \frac{\mathrm{d}\log p}{\mathrm{d}\log z} \bigg/ \frac{\mathrm{d}\log q}{\mathrm{d}\log z} = \frac{\mathrm{Cov}[\log z, \log p^{**}]}{\mathrm{Cov}[\log z, \log q^{**}]} \tag{9}$$

where p^{**} and q^{**} are the optimal *ex post* prices and quantities that the firm would set with full information:

$$p^{**} = \frac{\eta}{\eta-1}\mathcal{M}P \qquad \text{and} \qquad q^{**} = \left(\frac{\eta}{\eta-1}\right)^{-\eta} \frac{z}{(\mathcal{M}P)^{\eta}} \tag{10}$$

An econometric metaphor illustrates why this is the optimal way to set α_1. By Equation 9, the firm's optimal policy is equivalent to running the following two-stage least squares (2SLS) regression: the firm estimates how its optimal price should change with its optimal quantity, using the nominal demand state z as an instrument for the optimal quantity. The supply function is steep ($|\alpha_1|$ is large) if nominal demand predicts large movements in the *ex post* optimal price. In the 2SLS metaphor, this corresponds to a large coefficient in the "reduced form" regression of p^{**} on z. The supply function is flat ($|\alpha_1|$ is small) if nominal demand predicts large movements in the *ex post* optimal quantity. In the 2SLS metaphor, this corresponds to a large coefficient in the "first stage" regression of q^{**} on z.

The Effects of Uncertainty. A critical determinant of the firm's optimal responsiveness of prices to quantities is their relative uncertainty about the price level, real marginal costs, and demand. To build intuition for this, we first focus on the case in which the firm's supply schedule is upward-sloping. This occurs if $0 \leq \mathrm{Cov}[\log z, \log(\mathcal{M}P)] \leq \frac{1}{\eta}\mathrm{Var}[\log z]$: high demand predicts that nominal costs are higher, but not too much higher. In this case, greater price-level uncertainty (σ_P^2 increases) steepens the optimal supply schedule. Intuitively, not knowing the prices of your competitors makes more aggressive dynamic pricing (*i.e.*, a strategy closer to quantity-setting) attractive because this allows one's *relative* price to adjust *ex post*. On the other hand, greater demand uncertainty (σ_Ψ^2 increases) flattens the optimal supply schedule. Intuitively, demand uncertainty favors a strategy closer to a fixed price as it allows production to adjust to accommodate greater demand. Finally, greater covariances between real marginal costs and demand and real marginal costs and the price level increase the firm's inverse supply elasticity. Intuitively, when these covariances increase, the firm expects to produce more exactly when it is more costly. Thus, the firm optimally sets a steeper supply schedule to avoid over-producing in response to changes in demand.

We finally observe that a positively sloped supply function is not guaranteed: if nominal costs move sufficiently with nominal demand, then a monopolist may prefer a *downward* sloping supply function in order to hedge against high costs in high-demand states.

The Effect of Market Power. The elasticity of demand plays two roles in determining the optimal (inverse) elasticity of supply. The first relates to *payoffs*: when η is high, *ex post* optimal quantities are more sensitive to changes in nominal marginal costs (holding fixed nominal demand). Intuitively, when goods are more substitutable, the firm's optimal policy depends dramatically on whether its marginal costs are above or below others' prices. The second role relates to *information*: when η is high, nominal demand contains relatively more information about the price level P and less about real demand Ψ.

In general, the interaction of these two forces can make the optimal supply function steepen or flatten when η increases. Below, we describe an intuitive sufficient condition under which a greater elasticity of demand induces steeper supply:

Corollary 1 (The Elasticity of Demand and Optimal Supply). *A sufficient condition for greater market power to lower the inverse supply elasticity, or* $\frac{\partial \alpha_1}{\partial \eta} > 0$, *is that each of the following three inequalities holds:*

$$\alpha_1 \geq 0,\ \sigma_{\mathcal{M},P} \geq 0,\ 2\eta\sigma_{\mathcal{M},P} + \sigma_{\mathcal{M},\Psi} \geq \sigma_{P,\Psi} \tag{11}$$

Proof. See Appendix A.2. □

The force of these conditions is to restrict the extent to which high nominal demand predicts low marginal costs. In this case, the dominant logic is the following: when demand is highly sensitive to relative prices, an upward-sloping aggregate supply function better allows a firm to index its prices relative to its nominal costs. As discussed earlier, this allows the firm to better hedge its risks from setting the "wrong" price. Later, in our quantitative analysis (Section 5), we find that the condition of Corollary 1 always holds in US data since 1960 as long as $\eta > 2.5$. Thus, the empirically relevant case appears to be that a lower elasticity of demand flattens firms' optimal supply function.

Pure Price- and Quantity-Setting Obtain in Extreme Limits. The previous result makes clear that pure price-setting and quantity-setting are two isolated points in the larger space of supply functions. Moreover, they are almost never optimal. We observe below that they are obtained in the limiting cases of extreme demand or price-level uncertainty:

Corollary 2 (A Foundation for Price-Setting and Quantity-Setting). *The following statements are true:*

1. *As* $\sigma_P^2 \to \infty$, $|\alpha_1| \to \infty$ *and the optimal plan converges to quantity-setting.*
2. *As* $\sigma_\Psi^2 \to \infty$, $\alpha_1 \to 0$ *and the optimal plan converges to price-setting.*

Thus, focusing on price- and quantity-setting is justified when *and only when* one source of risk is dominant. In a macroeconomic environment, however, we may expect all sources of risk to be present in comparable orders of magnitude. In such a scenario, the extreme policies may perform poorly, for both the firm and the economic analyst.

2.3 Supply Functions in Other Environments

We have made many specific assumptions on technology and demand for exposition simplicity. As we will show in the remainder of the paper, these same assumptions will also allow us to tractably study a general-equilibrium environment with an endogenous feedback loop between supply-function choice and endogenous uncertainty.

Nonetheless, the basic economic logic of supply-function choice extends to a much broader class of models. The unifying theme is the observation that choosing supply functions is tantamount to incorporating information regarding the desired price and quantity from market clearing. This basic observation did not rely on our specific description of technology and demand. As such, firms' optimal supply functions in other environments inherit the following fundamental logic of Theorem 1: firms want prices to increase steeply in quantities when high demand predicts a high desired price but not a high desired quantity. Moreover, taking the perspective of supply functions versus the traditional view of price- or quantity-setting introduces novel channels through which firms' uncertainty shapes their choices.

In Appendix B, we illustrate this point by characterizing optimal supply functions in four extensions that explore different assumptions about real and nominal rigidities. We briefly summarize all four extensions below.

Multiple Inputs, Decreasing Returns-to-scale, and Monopsony. First, we allow for a Cobb-Douglas production technology with multiple inputs, decreasing returns to scale, and convex costs of hiring additional inputs (capturing monopsony). These forces change the analysis solely by introducing a single composite parameter that aggregates the decreasing returns and monopsony forces across inputs. We show in Proposition 4 that the optimal supply function remains optimally log-linear and uncertainty enters in a similar way. Decreasing returns to scale and monopsony power (that may arise because of adjustment costs in production, for instance) both reduce the optimal supply elasticity of the firm and push the firm toward more rigid quantities.

Endogenous Markups. Second, we allow for demand that is not iso-elastic in a class that separates the firm's own-price elasticity of demand from the firm's cross-price elasticity of demand. We solve for the optimal supply curve in this case in Proposition 5. We show that uncertainty enters in a similar way but the optimal supply curve ceases to be log-linear as the optimal markup is endogenous to the scale of production. Intuitively, this allows the model to capture the possibility that "goods flying off the shelves" is informative about the desired markup.

Additional Choice Variables. Third, we allow for the firm to choose additional variables beyond prices and quantities that may affect the joint distribution of all variables relevant to the firm, such as marginal costs and demand. One example is a firm that can invest in improving the quality of its product. We show in Proposition 6 that the firm's supply function remains optimally log-linear but with a slope that depends on the uncertainty that is induced by their choice of additional variables. Using this, we characterize the value of any choice of additional variables and show in an example how such a firm would optimally choose the quality of its product at a cost.

Sticky Prices. Fourth, we enrich the firm's problem with price stickiness as in Calvo (1983) to capture the empirically relevant possibility that the firm's prices may be fixed for multiple periods (Bils and Klenow, 2004). We show in Proposition 7 that the firm's supply function remains optimally log-linear but with a slope that depends on how the firm learns from its demand today about the full future sequence of its nominal marginal costs. This demonstrates the simplicity with which the supply function approach could be integrated into macroeconomic models with sticky prices, like the textbook New Keynesian model (Woodford, 2003a).

3 Supply Functions in a Macroeconomic Model

We now embed supply-function choice in a monetary macroeconomic model. We otherwise use intentionally standard microfoundations (see, *e.g.*, Woodford, 2003b; Hellwig and Venkateswaran, 2009). These microfoundations will allow for a closed-form analysis and highlight the core economics of supply functions without any approximations. In this context, we will be interested in understanding three things: (i) how the microeconomic inverse supply elasticity maps into the elasticity of aggregate supply, (ii) how equilibrium macroeconomic dynamics endogenously influence the optimal microeconomic supply elasticity, and (iii) how these two channels interact to determine equilibrium macroeconomic dynamics.

3.1 Households

Time is discrete and infinite $t \in \mathbb{N}$. There is a continuum of differentiated goods indexed by $i \in [0, 1]$, each of which is produced by a different firm.

A representative household has standard (Hellwig and Venkateswaran, 2009; Golosov and Lucas, 2007) expected discounted utility preferences with discount factor $\beta \in (0,1)$ and per-period utility defined over consumption of each variety, C_{it}; holdings of real money balances, $\frac{M_t}{P_t}$; and labor effort supplied to each firm, N_{it}:

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left(\frac{C_t^{1-\gamma}}{1-\gamma}+\ln\frac{M_t}{P_t}-\int_{[0,1]}\phi_{it}N_{it}\,\mathrm{d}i\right)\right] \tag{12}$$

where $\gamma \geq 0$ indexes income effects in both money demand and labor supply and $\phi_{it} > 0$ is the marginal disutility of labor supplied to firm i at time t, which is an IID lognormal variable with time-dependent variance, or $\log\phi_{it} \sim N(\mu_\phi, \sigma_{\phi,t}^2)$. The consumption aggregate C_t is a constant-elasticity-of-substitution aggregate of the individual consumption varieties with elasticity of substitution $\eta > 1$:

$$C_t = \left(\int_{[0,1]}\vartheta_{it}^{\frac{1}{\eta}}c_{it}^{\frac{\eta-1}{\eta}}\,\mathrm{d}i\right)^{\frac{\eta}{\eta-1}} \tag{13}$$

where ϑ_{it} is an IID preference shock that is also lognormal with time-dependent variance, or $\log\vartheta_{it} \sim N(\mu_\vartheta, \sigma_{\vartheta,t}^2)$. We also define the corresponding ideal price index:

$$P_t = \left(\int_{[0,1]}\vartheta_{it}p_{it}^{1-\eta}\,\mathrm{d}i\right)^{\frac{1}{1-\eta}} \tag{14}$$

Households can save in either money or risk-free one-period bonds B_t (in zero net supply) that pay an interest rate of $(1+i_t)$. The household owns the firms in the economy, each of which has profits of Π_{it}. Thus, the household faces the following budget constraint at each time t:

$$M_t + B_t + \int_{[0,1]}p_{it}C_{it}\,\mathrm{d}i = M_{t-1} + (1+i_{t-1})B_{t-1} + \int_{[0,1]}w_{it}N_{it}\,\mathrm{d}i + \int_{[0,1]}\Pi_{it}\,\mathrm{d}i \tag{15}$$

where p_{it} is the price of variety of variety i and w_{it} is a variety-specific nominal wage.

The aggregate money supply follows an exogenous random walk with drift μ_M and time-dependent volatility σ_t^M:

$$\log M_t = \log M_{t-1} + \mu_M + \sigma_t^M\varepsilon_t^M \tag{16}$$

where the monetary innovation is an IID random variable that follows $\varepsilon_t^M \sim N(0,1)$. So that interest rates remain strictly positive, we assume that $\frac{1}{2}(\sigma_t^M)^2 \leq \mu_M$ for all $t \in \mathbb{N}$.

3.2 Firms

The production side of the model follows closely the model from Section 2. Each consumption variety is produced by a separate monopolistic firm, also indexed by $i \in [0,1]$. Each firm operates a production technology that is linear in labor:

$$q_{it} = \zeta_{it} A_t L_{it} \tag{17}$$

where L_{it} is the amount of labor employed, ζ_{it} is IID lognormal with time-dependent volatility $\sigma_{\zeta,t}$, or $\log \zeta_{it} \sim N(\mu_\zeta, \sigma^2_{\zeta,t})$, and $\log A_t$ follows an AR(1) with time-varying volatility σ^A_t:

$$\log A_t = \rho \log A_{t-1} + \sigma^A_t \varepsilon^A_t \tag{18}$$

where the productivity innovations are IID and follow $\varepsilon^A_t \sim N(0,1)$. For our main analysis, we assume that innovations to aggregate productivity and the money supply are independent. In Appendix C, we generalize our results to allow for arbitrary correlation between these shocks. When the firm sells output at price p_{it} and hires labor at wage w_{it}, its nominal profits are given by $\Pi_{it} = p_{it} q_{it} - w_{it} L_{it}$. Since firms are owned by the representative household, their objective is to maximize expectations of real profits, discounted by a real stochastic discount factor Λ_t. Thus, the firm's payoff is $\frac{\Lambda_t}{P_t}\Pi_{it}$.

At the beginning of time period t, firms first observe A_{t-1} and M_{t-1}. Firms also receive private signals about aggregate productivity s^A_{it} and the money supply s^M_{it}:

$$\begin{aligned} s^A_{it} &= \log A_t + \sigma_{A,s,t} \varepsilon^{s,A}_{it} \\ s^M_{it} &= \log M_t + \sigma_{M,s,t} \varepsilon^{s,M}_{it} \end{aligned} \tag{19}$$

where the signal noise is IID and follows $\varepsilon^{s,A}_{it}, \varepsilon^{s,M}_{it} \sim N(0,1)$. Firms are uncertain about the idiosyncratic productivity shock ζ_{it}, demand shock ϑ_{it}, and labor supply shock ϕ_{it}.[3]

3.3 Markets and Equilibrium

In each period, conditional on the aforementioned information set, firms choose a supply function. As in Section 2, firms make this decision under uncertainty about demand, costs, and the stochastic discount factor. But, as will become clear, this uncertainty is now partially about *endogenous* objects. After firms make their choices, the money supply, idiosyncratic demand shocks, and both aggregate and idiosyncratic productivity are realized. Finally, the

[3] It is not important that firms are fully uninformed about these quantities. The model's predictions would be identical if firms also received noisy signals about their idiosyncratic shocks.

household makes its consumption and savings decisions and any prices that were not fixed adjust to clear the market. Formally, we define an equilibrium as follows:

Definition 1 (Supply-Function General Equilibrium). *An equilibrium is a collection of variables*

$$\left\{\{p_{it}, q_{it}, C_{it}, N_{it}, L_{it}, w_{it}, \Pi_{it}\}_{i\in[0,1]}, C_t, P_t, M_t, A_t, B_t, N_t, \Lambda_t\right\}_{t\in\mathbb{N}}$$

and a sequence of supply functions $\left\{f_{it} : \mathbb{R}^2_{++} \to \mathbb{R}\right\}_{i\in[0,1],t\in\mathbb{N}}$ *such that, in all periods:*

1. *All firms choose their supply function f_{it} to maximize expected real profits under the household's stochastic discount factor.*
2. *The household chooses consumption C_{it}, labor supply N_{it}, money holdings M_t, and bond holdings B_t to maximize their expected utility subject to their lifetime budget constraint, while Λ_t is the household's marginal utility of consumption.*
3. *Money supply M_t and productivity A_t evolve exogenously via Equations 16 and 18.*
4. *Firms' and consumers' expectations are consistent with the equilibrium law of motion.*
5. *The markets for the intermediate goods, final good, labor varieties, bonds, and money balances all clear.*

We will also often be interested in describing equilibrium dynamics conditional on a (potentially suboptimal) supply function for firms. Formally, these *temporary equilibria* are equilibria in which we do not require statement (1) of Definition 1.

4 Supply Function Choice and Aggregate Supply

We now study the model's equilibrium predictions, focusing on the equilibrium determination of the aggregate supply curve. We proceed in three steps. First, we solve for all equilibrium conditions except for the firm's supply-function decision. Second, we show that, fixing any log-linear supply schedule, the economy admits a unique log-linear equilibrium that has a simple Aggregate Supply and Aggregate Demand representation. The slope of aggregate supply depends on the slope of firm-level supply, in conjunction with other parameters. Third, we combine this with our solution for optimal supply schedules from Theorem 1 and fully characterize equilibrium in terms of a single, scalar fixed-point equation for the firm-level supply elasticity. We study how strategic interactions, the elasticity of demand, and the combination of microeconomic demand uncertainty alongside aggregate productivity and monetary uncertainty affect the equilibrium aggregate supply elasticity. Finally, we show how supply function choice can be tractably incorporated in a larger class of dynamic general equilibrium models.

4.1 Firms' Uncertainty in Equilibrium

We begin by deriving the general-equilibrium analogs of the four objects that were central to the firm's problem in Section 2: firm-specific demand shocks, firm-specific marginal costs, the price level, and the stochastic discount factor. We do so by deriving the household's Euler equations for bonds, money, and labor supply. We summarize the results of this below:

Proposition 1 (Firm-Level Shocks in General Equilibrium)**.** *In any temporary equilibrium, demand shocks, aggregate price shocks, stochastic discount factor shocks, and marginal cost shocks follow:*

$$\Psi_{it} = \vartheta_{it} C_t \,, \quad P_t = \frac{i_t}{1+i_t} C_t^{-\gamma} M_t \,, \quad \Lambda_t = C_t^{-\gamma} \,, \quad \mathcal{M}_{it} = \frac{\phi_{it} C_t^{\gamma}}{z_{it} A_t} \tag{20}$$

where i_t is a deterministic function of only exogenous parameters that we provide in the Appendix.

Proof. See Appendix A.3. □

Each of these expressions is intuitive given the general equilibrium structure of the model. First, the firm's demand shock is the product between its idiosyncratic demand shock and aggregate demand. Second, the demand for real money balances is decreasing in the interest rate as this determines the opportunity cost of holding money (which itself depends on the future path of monetary volatility, the drift of the money supply, and the household's discount factor). Moreover, this demand is increasing in the household's level of consumption because of an income effect, which is governed by the curvature of consumption utility γ. Intuitively, when consumption utility has greater curvature, income effects in money demand are larger and money demand is more responsive to changes in consumption. Thus, consumption responds less to real money balances when γ is large. Third, the SDF is the marginal utility of consumption. Finally, the real marginal cost of firms is increasing in the level of consumption because of the same income effect, and decreasing in their productivity.

The uncertainty the firm faces in light of Proposition 1 concerns endogenous objects. This introduces strategic uncertainty (*i.e.*, payoff-relevant uncertainty about other firms' choices).[4] Moreover, firms' uncertainty is correlated across variables due to macroeconomic linkages in the product, money, and labor markets.

An important technical implication of Proposition 1 is that, if C_t is log-normal, then so too is $(\Psi_{it}, P_t, \Lambda_t, \mathcal{M}_{it})$. This follows from the fact that all four expressions are log-linear

[4] One interesting implication of Proposition 1 is that nominal wages, $w_{it} = \frac{i_t}{1+i_t} \phi_{it} M_t$, provide information *only* about exogenous objects. A stronger implication is that a model in which firms draw inferences from both output-market prices and input-market prices has identical predictions to our studied model.

and all other fundamentals $(A_t, M_t, \vartheta_{it}, \phi_{it}, z_{it})$ are log-normal by assumption. Therefore, if we can find that C_t is log-normal in equilibrium, our Theorem 1 can be directly applied to determine the optimal supply function in general equilibrium in our fully non-linear setting. We will call an equilibrium in which $\log C_t$ is linear in $(\log A_t, \log M_t)$ a *log-linear equilibrium.*

4.2 From Supply to Aggregate Supply with Fixed Functions

We start by assuming that firms' exogenously set log-linear supply functions:

$$\log p_{it} = \alpha^*_{0t,i}(\alpha_{1,t}) + \alpha_{1,t} \log q_{it} \tag{21}$$

where $\alpha_{1,t} \in \mathbb{R}$ is a fixed parameter and $\alpha^*_{0t,i}(\alpha_{1,t})$ is the profit-maximizing "intercept" conditional on this slope.[5] This optimal intercept depends on the slope $\alpha_{1,t}$, the firm's beliefs, and realized demand, but not (independently) on the realized quantity. This has two purposes. First, this assumption allows us to explore what happens in temporary equilibrium when firms use a given supply function. This is useful for understanding what strategic restrictions on firms' pricing strategies (*e.g.,* exogenously imposing price-setting) imply for macroeconomic dynamics. Second, this assumption is our guess about what firms' supply function will be in equilibrium, which we will later verify as correct. This allows us to understand the ultimate macroeconomic implications of optimal supply function choice.

Conditional on these supply functions, we guess and verify that there exists an equilibrium in which aggregate consumption and the price level are log-linear in aggregate shocks:

$$\begin{aligned} \log P_t &= \chi_{0,t}(\alpha_{1,t}) + \chi_{A,t}(\alpha_{1,t}) \log A_t + \chi_{M,t}(\alpha_{1,t}) \log M_t \\ \log C_t &= \tilde{\chi}_{0,t}(\alpha_{1,t}) + \tilde{\chi}_{A,t}(\alpha_{1,t}) \log A_t + \tilde{\chi}_{M,t}(\alpha_{1,t}) \log M_t \end{aligned} \tag{22}$$

To this end, we define the posterior weight on firms' signals of productivity and the aggregate money supply as, respectively, $\kappa_t^A = \left(1 + \left(\sigma_{A,s,t}/\sigma_t^A\right)^2\right)^{-1}$ and $\kappa_t^M = \left(1 + \left(\sigma_{M,s,t}/\sigma_t^M\right)^2\right)^{-1}$. Moreover, define the slope of supply functions in terms of $\log z_{it} = \eta \log P_t + \log \Psi_{it}$ as:[6]

$$\omega_{1,t} = \frac{\alpha_{1,t}}{1 + \eta \alpha_{1,t}} \tag{23}$$

We now characterize equilibrium macroeconomic dynamics with fixed supply functions. We show that macroeconomic dynamics in log-linear general equilibrium are equivalent to

[5] We will later verify that all firms use a common slope in equilibrium. In light of Theorem 1, this is because all firms are exposed to uncertainty in the same way.

[6] So everything remains well defined, we will impose that $\omega_{1,t} \neq (\eta - 1/\gamma)(1 - \kappa_t^x)$ for $x \in \{A, M\}$. Our analysis verifies that these values of $\omega_{1,t}$ cannot occur in log-linear equilibrium (see the proof of Theorem 3).

those that would be generated by an Aggregate Demand and Aggregate Supply (AD/AS) model, in which productivity shocks shift the AS curve and money shocks shift the AD curve. Critically, the slope of aggregate supply depends on the slope of firms' supply schedules.

Theorem 2 (Equilibrium and AD/AS Representation)**.** *There is a unique log-linear temporary equilibrium. The behavior of aggregate prices and output in this temporary equilibrium is equivalent to that generated by the following "Aggregate Demand/Aggregate Supply" model:*

$$\log P_t = \log\left(\frac{i_t}{1+i_t}\right) - \epsilon_t^D \log Y_t + \log M_t \tag{AD}$$

$$\log P_t = \log \bar{P}_t + \epsilon_t^S \log Y_t + \delta_t \log A_t \tag{AS}$$

where the inverse supply and demand elasticities are given by:

$$\epsilon_t^S = \gamma \frac{\kappa_t^M + \frac{\omega_{1,t}}{\gamma}(1-\kappa_t^M)}{(1-\omega_{1,t}\eta)(1-\kappa_t^M)} \quad \textit{and} \quad \epsilon_t^D = \gamma \tag{24}$$

and the interest rate i_t, the intercept for the price level $\log \bar{P}_t$, and the partial equilibrium effect of productivity shocks δ_t do not depend on $(\log P_t, \log Y_t, \log M_t, \log A_t)$.[7]

Proof. See Appendix A.4. □

In this representation, the aggregate demand curve combines the Euler equations for money and bonds with the transversality condition and implies that: (i) the interest rate is a function of exogenous parameters and (ii) aggregate consumption has an elasticity of $1/\gamma$ to changes in real money balances. The slope (or inverse elasticity) of aggregate demand in our model is γ. The aggregate supply curve describes the equilibrium relationship between aggregate output and aggregate prices by aggregating firms' microeconomic pricing and production decisions conditional on a fixed inverse supply elasticity.

We illustrate this representation in Figure 2. An "aggregate demand shock," an increase of the money supply by $\log M_1 - \log M_0 = \Delta \log M > 0$, shifts up the AD curve. This has an effect of $\frac{\Delta \log M}{\epsilon^D + \epsilon^S}$ on real output and $\epsilon^S \frac{\Delta \log M}{\epsilon^D + \epsilon^S}$ on the price level. The price effect is larger and the quantity effect is smaller if ϵ^S is large. This calculation also makes clear that ϵ^S is the relative effect of an aggregate demand shock on the price level versus real output.

The Propagation of Demand Shocks. To obtain more intuition for the propagation of shocks via firms' supply schedules, we expand the response of the price level to a money shock

[7]See Appendix A.4 for explicit formulae for these terms.

into a partial equilibrium effect and a series of higher-order general equilibrium effects:[8]

$$\frac{\Delta \log P}{\Delta \log M} = \frac{\epsilon_t^S}{\epsilon_t^D + \epsilon_t^S} = \underbrace{\left(\kappa_t^M + \frac{\omega_{1,t}}{\gamma}(1-\kappa_t^M)\right)}_{\text{Partial Equilibrium}} \times \underbrace{\sum_{j=0}^{\infty}\left(\omega_{1,t}\left(\eta - \frac{1}{\gamma}\right)(1-\kappa_t^M)\right)^j}_{\text{General Equilibrium}} \tag{25}$$

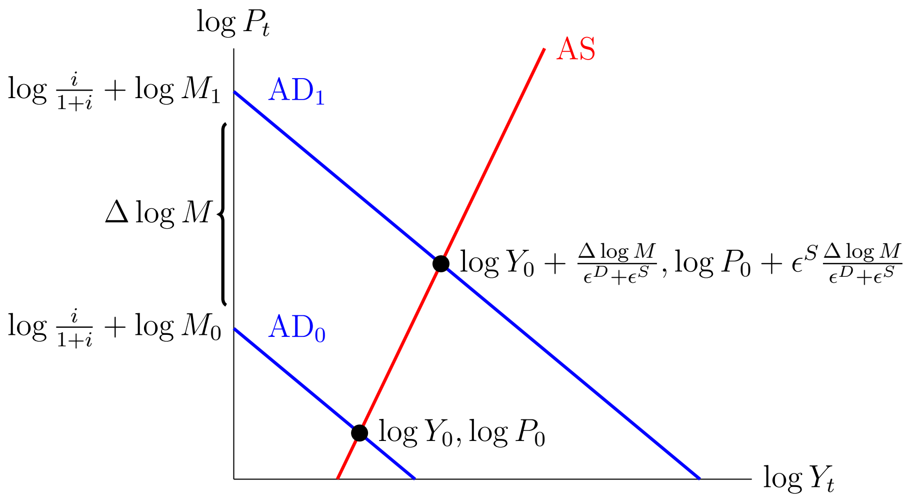
It is first helpful to understand the case of pure price-setting ($\omega_{1,t} = 0$). In this case, an increase in M by 1% raises real money balances by 1%, which increases consumption demand by $1/\gamma$%. From the households' labor supply condition, this increases real marginal costs by $\gamma \times 1/\gamma = 1$%. As the firm wishes to set its relative price equal to a constant mark-up over real marginal costs, the direct partial equilibrium effect of the shock is for the firms to increase prices by 1%. Next, in general equilibrium (*i.e.*, accounting for the effect of the shock on the aggregate price level), the 1% increase in the price level induces a given firm to increase prices by 1%. However, higher prices also reduce real consumption demand, which lowers marginal costs by $\gamma \times 1/\gamma$%. Thus, the general equilibrium effects net out, and an increase in M increases prices one-for-one. Finally, when firms are imperfectly informed of the money supply, they perceive that real marginal costs increase by only κ_t^M% on average. Thus, the price level increases by κ_t^M%, which is obtained in Equation 25 by setting $\omega_{1,t} = 0$.

Consider now the case with general supply functions. A 1% increase in the money supply induces the firm to experience a $1/\gamma$% demand shock. As the firm has an inverse supply elasticity of $\omega_{1,t}$, this leads to an additional effect in which the firm increases prices by $\omega_{1,t}/\gamma$%. In order to keep its mark-up over real marginal costs fixed, firms reduce prices by $\kappa_t^M \times \omega_{1,t}/\gamma$% on average (via their intercept), which is their perceived increase in their demand following an increase in the money supply. Thus, the partial equilibrium (PE) effect involves an addition term relative to the case of pure price setting, given by $\omega_{1,t}(1-\kappa_t^M)/\gamma$.

With supply functions, there are also general equilibrium effects: a 1% in the aggregate price level leads all firms to experience a demand shock of η% (as the prices of their competitors have increased). But it also reduces real consumption demand by $1/\gamma$%, since real money balances fall. Together, these effects lead firms to increase their prices by $\omega_{1,t} \times (\eta - 1/\gamma)$%. To keep the markup over their nominal marginal cost constant, firms reduce prices by $\kappa_t^M \times \omega_{1,t} \times (\eta - 1/\gamma)$% on average, which is their perceived increase in their demand. In total, out of a monetarily induced 1% increase in the aggregate price level, the average increase in firms' prices is therefore $\omega_{1,t} \times (\eta - 1/\gamma) \times (1-\kappa_t^M)$%. Iterating this to

[8]This expression is derived by multiplying the numerator and denominator of $\epsilon_t^S/(\epsilon_t^D + \epsilon_t^S)$ by $(1-\omega_{1,t}\eta)(1-\kappa_t^M)$ and expanding it into a geometric summation. The summation only converges when $\left|\omega_{1,t}(\eta - 1/\gamma)(1-\kappa_t^M)\right| < 1$. Our fixed point arguments establish that the claimed formulae hold more generally whenever $\left|\omega_{1,t}(\eta - 1/\gamma)(1-\kappa_t^M)\right| \neq 1$. The proof of Theorem 3 shows the final case of $\left|\omega_{1,t}(\eta - 1/\gamma)(1-\kappa_t^M)\right| = 1$ cannot happen in equilibrium.

Figure 2: An Aggregate Supply and Demand Representation



Notes: An aggregate supply and demand illustration of dynamics after a shock of size $\Delta \log M$ to the money supply (see Theorem 2).

all subsequent price increases in GE yields Equation 25.

A novel implication of our model is that the extent of general-equilibrium strategic complementarity hinges critically on the slope of the supply function. Starkly, general-equilibrium interactions would be entirely absent (*i.e.,* pricing decisions would be neither complements nor substitutes) if price-setting ($\omega_{1,t} = 0$) were exogenously assumed: the PE effect would be $\kappa_t^M\%$ and the GE effect would be 0%. Through this lens, predictions for complementarity in benchmark price-setting (Woodford, 2003a) and quantity-setting (Angeletos and La'O, 2010) models are joint predictions of the economic environment and an exogenous restriction on firms' strategy space.

The Propagation of Supply Shocks. While our study is primarily focused on predictions for the aggregate supply curve and transmission of demand shocks, our model also makes predictions for the transmission of supply shocks. In the AD/AS representation, a positive shock to $\log A_t$ corresponds to an outward shift of the AS curve, which raises real output and lowers the price level. While the relative effect on the price level and on real output is $\epsilon^D = \gamma$, the level of these responses varies with the slope of supply functions, $\omega_{1,t}$.

To understand the reason for this, we can, just as above, decompose the effect into partial and general equilibrium components:

$$\frac{\Delta \log P}{\Delta \log A} = \underbrace{-\kappa_t^A}_{\text{PE}} \times \underbrace{\sum_{j=0}^{\infty} \left(\omega_{1,t} \left(\eta - \frac{1}{\gamma} \right) (1 - \kappa_t^A) \right)^j}_{\text{GE}} \tag{26}$$

The PE effect is immediate: firms perceive a $\kappa_t^A\%$ decrease in their real marginal costs and adjust their prices by an equal percentage. The GE effects of the change in the price level are identical to those under monetary shocks, other than that productivity uncertainty may differ from monetary uncertainty. Thus, strategic interactions are attenuated by a factor of $1-\kappa_t^A$ rather than $1-\kappa_t^M$. In sum, a key takeaway from our analysis is that the general equilibrium transmission of shocks crucially depends on the slopes of microeconomic supply curves in the economy.

4.3 The Slope of Aggregate Supply in Temporary Equilibrium

We now study how various microeconomic forces affect the slope of aggregate supply.

Corollary 3 (How Microeconomic Forces Affect Aggregate Supply)**.** *If firms' supply curves are upward-sloping (i.e., $\omega_{1,t} \in [0, 1/\eta)$), then the following statements are true:*

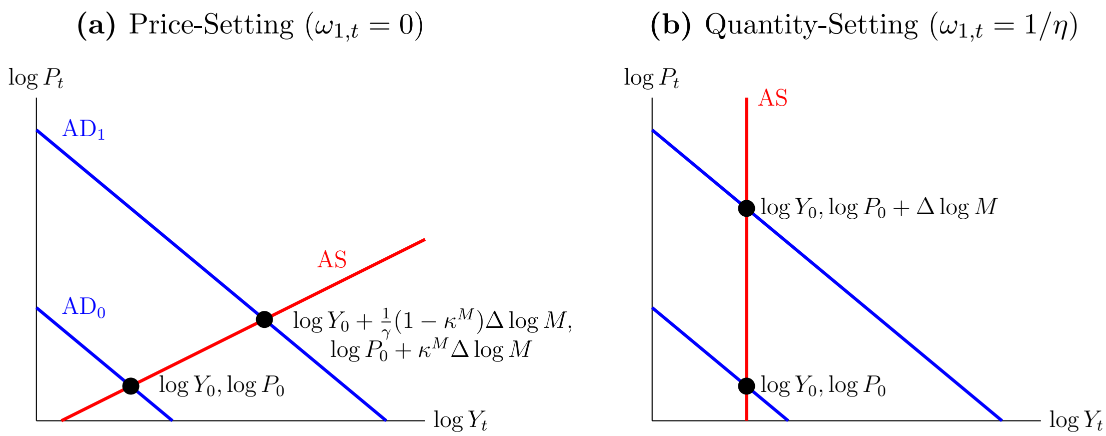
1. *Steeper microeconomic supply steepens the AS curve: $\partial\epsilon_t^S / \partial\omega_{1,t} \geq 0$.*
2. *Precision of private information about money steepens the AS curve: $\partial\epsilon_t^S / \partial\kappa_t^M \geq 0$.*
3. *Income effects steepen the AS curve: $\partial\epsilon_t^S / \partial\gamma \geq 0$.*
4. *A higher elasticity of demand steepens the AS curve: $\partial\epsilon_t^S / \partial\eta \geq 0$.*

Proof. Follows immediately from differentiation of Equation 24. □

To understand the first statement, observe that a steeper microeconomic supply function makes prices more responsive to realized quantities *ex post*. At the aggregate level, this implies that the price level is also more responsive to changes in output. Second, more precise private information about the money supply steepens the AS curve because firms respond to the perceived increase in the money supply by increasing *average* prices (as modulated through the intercept $\alpha_{0t,i}^*$). This reduces variation in real money balances, thereby attenuating the effect of demand shocks on aggregate output. Third, output responds less to money balances the higher is γ (see Proposition 1). Consequently, a higher γ steepens the AS curve.

Finally, a lower elasticity of demand flattens the AS curve. Crucially, this effect is non-zero if and only if $\omega_{1,t} \neq 0$, *i.e.*, firms do not undertake pure *price-setting*. This flattening operates through the general equilibrium transmission mechanisms of the model. When other firms raise their prices in response to a money supply shock, firm-level demand increases because the firm's *relative* price is now lower. The magnitude of this demand change is exactly parameterized by the elasticity of substitution η. If the responsiveness of prices to quantities at the firm level is non-zero, this demand increase generates an additional price level response. Consequently, higher market power flattens the AS curve by lowering the

Figure 3: Aggregate Supply Under Price-Setting and Quantity-Setting



Notes: An aggregate supply and demand illustration of dynamics after a shock of size $\Delta \log M$ to the money supply (see Theorem 2) under price-setting (panel a) and quantity-setting (panel b).

responsiveness of *firm-level* prices to *relative* price changes. This prediction is opposite to the prediction that Woodford (2003b) obtains: in a New Keynesian model with decreasing returns to scale, the slope of the Phillips curve is lower when demand is more elastic.[9]

Aggregate Supply Under Price-Setting and Quantity Setting. We can illustrate some of these effects even more sharply by describing the slope of aggregate supply under the common assumptions of pure price-setting and quantity-setting. We find that the aggregate supply curve is vertical under quantity-setting and maximally flat under price-setting:

Corollary 4 (Aggregate Supply Under Price- and Quantity-Setting)**.** *If firms engage in price-setting* ($\omega_{1,t} = 0$)*, then:*

$$\epsilon_t^S = \gamma \frac{\kappa_t^M}{1-\kappa_t^M} \tag{27}$$

If firms engage in quantity-setting ($\omega_{1,t} = \frac{1}{\eta}$)*, then:*

$$\epsilon_t^S = \infty \tag{28}$$

We illustrate these two "extreme" predictions for aggregate supply and demand in Figure 3. Since ϵ_t^S is increasing in $\omega_{1,t}$, the price-setting case provides a *lower bound* on the inverse elasticity of the aggregate supply curve (among all upward sloping supply functions) and therefore maximizes the real effects of demand shocks. Moreover, as mentioned above, the

[9] Moreover, the interaction between market power and the slope of aggregate supply arises for completely different reasons. In the New Keynesian model, the logic is that: when demand is very elastic, higher prices translate to much lower quantities and, under decreasing returns, much lower marginal costs. This dampens the desired price change in response to a nominal cost shock.

slope is invariant to the elasticity of demand only in this case. The case of price-setting recovers the aggregate supply elasticity of Lucas (1972) with the same insight that more precise information about the money supply leads to a steeper aggregate supply curve.

In sharp contrast, the AS curve is vertical under quantity-setting and money has *no real effects*. This is not a foregone conclusion, but an equilibrium result. Indeed, quantity-setting firms could condition their production on their monetary signal and money would have real effects if they did so. As a simple example, setting $\log q_{it} = s_{it}^M$ is feasible for firms and this would imply that money has real effects: $C_t \propto M_t$. The second part of Corollary 4 follows from the fact that if firms set quantities, then there is no equilibrium in which firms' quantities depend on the monetary signal.

These results emphasize that the kinds of strategies firms use have large macroeconomic consequences. It may be unappealing that the choice of the economic analyst about what kinds of strategies firms use has such large macroeconomic implications. A key benefit of the supply functions approach is that it allows the analyst to avoid imposing such restrictions and the potentially unintended consequences for macroeconomic predictions that follow.

4.4 The Equilibrium Slope of Aggregate Supply

We now endogenize the firm-level inverse supply elasticity as a best response to equilibrium macroeconomic dynamics. We have verified that if firms use log-linear supply functions, then aggregate dynamics are endogenously log-linear (by Theorem 2). Moreover, we have verified that if aggregate dynamics are log-linear, then firms' uncertainty is endogenously log-normal (by Proposition 1). Thus, we have shown that firms' supply curves are endogenously log-linear in a log-linear equilibrium (by Theorem 1). By combining these results, we reduce the determination of log-linear equilibrium in the full dynamic economy with functional supply decisions by firms to a single, scalar fixed-point equation for the slopes of supply functions:

Theorem 3 (Equilibrium Supply Elasticity Characterization)**.** *All (and only all) solutions $\omega_{1,t} \in \mathbb{R}$ of the following equation correspond to transformed inverse supply elasticities in log-linear equilibrium:*

$$\omega_{1,t} = T_t(\omega_{1,t}) \equiv \frac{\frac{\left(\eta-\frac{1}{\gamma}\right)\kappa_t^A}{1-\omega_{1,t}\left(\eta-\frac{1}{\gamma}\right)\left(1-\kappa_t^A\right)}(\sigma_{t|s}^A)^2 + \frac{\frac{1}{\gamma}+\left(\eta-\frac{1}{\gamma}\right)\kappa_t^M}{1-\omega_{1,t}\left(\eta-\frac{1}{\gamma}\right)\left(1-\kappa_t^M\right)}(\sigma_{t|s}^M)^2}{\sigma_{\vartheta,t}^2 + \left(\frac{\left(\eta-\frac{1}{\gamma}\right)\kappa_t^A}{1-\omega_{1,t}\left(\eta-\frac{1}{\gamma}\right)\left(1-\kappa_t^A\right)}\right)^2 (\sigma_{t|s}^A)^2 + \left(\frac{\frac{1}{\gamma}+\left(\eta-\frac{1}{\gamma}\right)\kappa_t^M}{1-\omega_{1,t}\left(\eta-\frac{1}{\gamma}\right)\left(1-\kappa_t^M\right)}\right)^2 (\sigma_{t|s}^M)^2} \tag{29}$$

where $\left(\sigma_{t|s}^A\right)^2 = (1-\kappa_t^A)\left(\sigma_t^A\right)^2$ *and* $\left(\sigma_{t|s}^M\right)^2 = (1-\kappa_t^M)\left(\sigma_t^M\right)^2$.

Proof. See Appendix A.5. □

This fixed-point equation incorporates the variances and covariances that enter the optimal supply function as a function of equilibrium macroeconomic dynamics when firms use supply functions with transformed inverse supply elasticities $\omega_{1,t}$. This depends on the responsiveness of aggregate prices and output to aggregate productivity and monetary shocks as well as the conditional uncertainty about these shocks when firms set their supply functions. Firms' idiosyncratic uncertainty about demand matters, but firms' uncertainty about idiosyncratic productivity and factor prices does not, as the variance of marginal costs *per se* does not matter for the choice of an optimal supply function.

This result makes clear that our model has different implications than those that study monetary non-neutrality with endogenous information acquisition. In our model, firms learn via the endogenous signals produced by the market mechanism. This differs from the premise of rational inattention models, wherein firms have unrestricted access to information but can only process it at a cost. This difference in information structures drives significant differences in results. In the rational inattention model of Maćkowiak and Wiederholt (2009), for example, *any* increase in idiosyncratic uncertainty lowers the responsiveness of prices to aggregate shocks. In our model, idiosyncratic productivity and cost uncertainty are not directly relevant for the slope of aggregate supply, whereas idiosyncratic demand uncertainty is directly relevant. Moreover, in our framework, the slope of aggregate supply depends on whether aggregate uncertainty is driven by real or nominal shocks. Thus, the information that arises endogenously through the market mechanism (firms' observation of their demand) is fundamentally different from the information that firms are restricted to obtain under existing models of costly information acquisition with price-setting firms.

In the remainder of this section, we will study this equation to understand equilibrium dynamics. First, we can use this result to establish log-linear equilibrium existence and provide a bound on the number of equilibria by rewriting the fixed-point equation as a quintic polynomial in $\omega_{1,t}$:

Proposition 2 (Existence and Number of Equilibria)**.** *There exists a log-linear equilibrium. There exist at most five log-linear equilibria.*

Proof. See Appendix A.6 □

To understand why there are possibly multiple equilibria, let us compare two situations: one where firms set flat supply functions and one where firms set steep supply functions. If firms set steep supply functions, then prices are highly sensitive to demand, and so the aggregate supply curve is steep (recall Theorem 2). Conversely, if firms set flat supply functions, then the aggregate supply curve is flat. When the economy is hit by larger monetary shocks than productivity shocks, an economy with a steeper aggregate supply curve will generate greater variability in the aggregate price level. Moreover, greater variability in the aggregate price level leads to large losses from price-setting behavior and thus leads to firms wanting to set steeper supply functions (recall Theorem 1). Because of this, there can exist a feedback mechanism whereby steeper supply functions can generate greater equilibrium price volatility, which in turn reinforces the benefits of setting a steeper supply function. If this feedback mechanism is sufficiently strong, then there may exist multiple equilibria. Appendix D.4 provides a quantitative illustration of the potential multiplicity of equilibria.

We now study how uncertainty, strategic interactions, and market power shape the aggregate supply elasticity in equilibrium.

A Simple Characterization Under Balanced Strategic Interactions. We first characterize the slope of aggregate supply under the parametric condition $\eta\gamma = 1$. Recall from our discussion in Section 4.2 that η parameterizes the strength of *strategic complementarities*: the additional increase in demand a firm faces from an increase in the aggregate price level due to a change in *relative* prices. In contrast, $1/\gamma$ parameterizes the strength of *strategic substitutabilities*: the reduction in demand a firm faces from an increase in the aggregate price level due to a reduction in *aggregate consumption* (that results from the reduction in real money balances). Hence, $\eta\gamma = 1$ corresponds to the case in which these forces exactly balance. This allows us to simplify the fixed point in Equation 29 considerably.

Corollary 5 (Idiosyncratic *vs.* Aggregate Demand Uncertainty)**.** *When $\eta\gamma = 1$, the unique inverse elasticity of aggregate supply is*

$$\epsilon_t^S = \gamma \frac{\kappa_t^M}{1-\kappa_t^M}\left(1 + \frac{1}{\gamma^2 \rho_t^2 \kappa_t^M}\right) \tag{30}$$

where $\rho_t = \frac{\sigma_{\vartheta,t}}{\sigma_{t|s}^M}$ is the relative uncertainty about demand vs. the money supply.

Proof. See Appendix A.7 □

First, observe that uncertainty about *aggregate* productivity does not enter the slope of aggregate supply when $\eta\gamma = 1$. This is because a perceived increase in aggregate productivity induces all firms to decrease their prices. In the absence of additional strategic interactions, firms will not respond to other firms' price reductions. Hence, the demand state z (Equation 8) is not useful for conducting inference about productivity and so κ_t^A does not enter the fixed point. The same is not true for uncertainty about the money supply, as it induces *direct* variation in the demand state z by changing aggregate consumption through real money balances. Consequently, firms can condition on the demand state z to learn about their nominal marginal costs when the money supply is uncertain.

Second, as $\rho_t \to \infty$, the inverse elasticity of aggregate supply approaches $\gamma \frac{\kappa_t^M}{1-\kappa_t^M}$. This is the AS curve slope under price-setting ($\omega_{1,t} = 0$). Intuitively, *idiosyncratic* demand conditions do not affect a given firm's marginal cost. Hence, as idiosyncratic demand becomes relatively more volatile, the firm optimally sets a constant price to keep its markup over marginal cost constant. Had the firm chosen $\omega_{1,t} \neq 0$, the firm would induce unprofitable variation in its price by responding to idiosyncratic demand conditions.

Third, as $\rho_t \to 0$, the inverse elasticity approaches infinity. Consequently, aggregate supply is perfectly inelastic and money has no real effects on output. This is the AS curve that arises from quantity-setting ($\omega_{1,t} = \frac{1}{\eta}$). Intuitively, as uncertainty about the money supply—and therefore the aggregate price level—increases, firms find it optimal to keep their quantities constant and let their relative price adjust to demand.

This discussion highlights that *relative* uncertainty about idiosyncratic *vs.* aggregate demand shocks is a crucial determinant of the slope of aggregate supply. Moreover, this feature only becomes relevant once firms are allowed to optimally choose their supply functions. As Corollary 4 demonstrates, if one were to exogenously impose price-setting or quantity-setting, the slope of aggregate supply is independent of any feature of idiosyncratic or aggregate uncertainty other than the signal-to-noise ratio for the money supply.

Thus, supply function choice implies, as a positive matter, a thorny trade-off for monetary policymakers. If the central bank wishes to maintain the discretion to surprise private agents via its policy actions, this will increase uncertainty about the money supply. In turn, this will steepen the equilibrium aggregate supply curve and make money less effective in guiding real economic outcomes. Therefore, maintaining monetary policy discretion may be, at least partially, self-defeating.

Equilibrium Under Dominant-Uncertainty Limits. To better understand how each source of uncertainty matters, we next characterize how equilibria behave as each source of uncertainty becomes dominant.[10] These results hold for any values of $\eta > 1$ and $\gamma > 0$, in contrast to the analysis above under balanced strategic interactions.

Corollary 6 (Dominant-Shock Limits)**.** *The following statements are true:*

1. *As $\sigma_{\vartheta,t} \to \infty$, in any equilibrium $\omega_{1,t} \to 0$ (price-setting)*
2. *As $\sigma_{t|s}^M \to \infty$, in any equilibrium $\omega_{1,t} \to \frac{1}{\eta}$ (quantity-setting)*
3. *As $\sigma_{t|s}^A \to \infty$ and $\eta\gamma \neq 1$, in any equilibrium $\omega_{1,t} \to \frac{1}{\eta - \frac{1}{\gamma}}$*

Proof. See Appendix A.8 □

[10] Formally, we take these limits for $\sigma_{t|s}^x$ and $x \in \{M, A\}$ by scaling $\sigma_{x,s,t}$ and σ_t^x by a common factor.

The intuition for this result mirrors that of Corollary 5. As idiosyncratic uncertainty about demand becomes dominant, firms find it optimal to set prices to keep their markup over real marginal costs constant. As prior uncertainty about the money supply becomes dominant, firms become more uncertain about the aggregate price level. Consequently, firms find it optimal to set quantities and let their relative prices adjust to meet demand. Finally, as uncertainty about aggregate productivity becomes dominant, firms use the demand state z to make inferences solely about the realization of aggregate productivity. Under perfect information, a 1% decrease in productivity would imply that firms raise their prices by 1%. This translates to an $(\eta - \frac{1}{\gamma})$% increase in demand for a given firm. Since firms believe that all fluctuations in demand are driven by productivity shocks, they set their optimal supply function slope to $\omega_{1,t} = \left[1/(\eta - \frac{1}{\gamma})\right]$. This ensures that a 1% increase in productivity will reduce prices by 1%, thus keeping their mark-up over nominal marginal costs constant. Observe that this force implies a downward-sloping supply curve whenever $\eta\gamma < 1$. Intuitively, if $\eta\gamma < 1$, income effects in labor supply are weak and the firm expects a lower real marginal cost after a positive demand shock.

The (Absent) Role of Total Uncertainty. We have so far seen that the nature of uncertainty (idiosyncratic *vs.* aggregate and demand *vs.* productivity) matters. Thus, the *presence* of uncertainty is of central importance to our analysis. However, a distinguishing feature of the theory that we have developed is that the total *level* of uncertainty does not matter. To make this claim formal, fix a scalar $\lambda \geq 0$ and scale all uncertainty in the economy according to:

$$(\sigma_{\vartheta,t}, \sigma_{z,t}, \sigma_{\phi,t}, \sigma_{A,t}, \sigma_{A,s,t}, \sigma_{M,t}, \sigma_{M,s,t}) \mapsto (\lambda\sigma_{\vartheta,t}, \lambda\sigma_{z,t}, \lambda\sigma_{\phi,t}, \lambda\sigma_{A,t}, \lambda\sigma_{A,s,t}, \lambda\sigma_{M,t}, \lambda\sigma_{M,s,t}) \quad (31)$$

In this sense, λ is a measure of the total level of uncertainty faced by firms. Define the correspondence $\mathcal{E}_t^S : \mathbb{R}_+ \rightrightarrows \bar{\mathbb{R}}$, where $\mathcal{E}_t^S(\lambda)$ is the set of equilibrium inverse supply elasticities for the level of uncertainty λ. We observe the following:

Proposition 3 (Invariance to Uncertainty and Discontinuity in the Limit)**.** *For $\lambda > 0$, $\mathcal{E}_t^S(\lambda)$ is constant and the equilibrium supply elasticity is invariant to the level of uncertainty. Moreover, $\mathcal{E}_t^S(0) = \{\infty\}$. Therefore, the equilibrium supply elasticity is discontinuous in the zero uncertainty limit:*

$$\lim_{\lambda \to 0} \mathcal{E}_t^S(\lambda) \neq \mathcal{E}_t^S(0) \quad (32)$$

Proof. See Appendix A.9 □

There are two important implications of this result. First, the total level of uncertainty does not matter for the slope of the aggregate supply curve. This constitutes a significant difference between our model and models with menu costs. Concretely, in menu cost models, any increase in uncertainty regarding the optimal reset price raises firms' private benefits of price flexibility without affecting the private costs, which are assumed to be fixed. Thus, increases in uncertainty lead to more variable prices at the micro level and more monetary neutrality at the macro level. By contrast, in our model, the level of uncertainty does not matter—only the relative magnitudes of uncertainty matter. One important implication of this difference is that idiosyncratic productivity uncertainty has *no effect* on the slope of aggregate supply in our model, while it would steepen aggregate supply in menu cost models that share our primitive economic assumptions on preferences and technology (such as Golosov and Lucas, 2007). Another important implication is that while idiosyncratic demand variation *flattens* aggregate supply in our model, it would have no effect in these menu cost models.

Second, the slope of the aggregate supply curve is discontinuous in the zero uncertainty limit. Indeed, $\mathcal{E}_t^S(\lambda)$ is typically neither upper hemi-continuous nor lower hemi-continuous at $\lambda = 0$. Thus, even a *vanishingly* small level of uncertainty can have significant effects on firm and aggregate behavior. This again represents a substantial difference to menu cost models, in which a small level of uncertainty has small effects on aggregate behavior and not the discontinuity that our model generates.[11] Importantly, this means that even in environments with low levels of uncertainty, the economic mechanisms that underlie our analysis are unchanged.

4.5 A General Framework for Macroeconomic Analysis

Our preceding analysis tractably illustrated the effect of supply functions in a fully non-linear fashion. To do so, we made a number of simplifying assumptions on utility and the nature of firms' production functions. However, we emphasize that our analysis can readily be extended to general linearized macroeconomic environments of the kind that are commonly studied in both state-of-the-art theoretical and quantitative work (see *e.g.,* McKay and Wolf, 2023). We now describe a general class of models in which the study of supply functions is tractable. We note that this is not meant as being exhaustive of the set of models in which supply functions are tractable or reasonable macroeconomic models.

Consider a model which generates a demand function for products given by $q_{i,t} = d(p_{it}, z_{it}^D)$, where the random variable z_{it}^D can depend on other, potentially endogenous variables of the model as well as exogenous stochastic processes. Assume further, in this dynamic setting, that the forward-looking value function V that the firm derives from setting a price

[11] Similarly, models with information acquisition and nominal rigidities (Afrouzi et al., 2024) are also different from our model in that they do not feature this discontinuity in the limit.

p_{it} and selling a quantity q_{it} is given by $V(p_{it}, q_{it}, \mathbf{z}_{it}^V)$, where $\mathbf{z}_{it}^V$ is an n_V-sized vector of (potentially endogenous) variables that affect firm's value at time t. Candidate modifications to our framework that could be incorporated in this fashion include decreasing returns-to-scale, monopsony, endogenous markups, price stickiness, investments, and endogenous quality choice (see Appendix B).

As in our model from Section 2, the firm's optimal supply function problem is to choose a price that is contingent on demand q_{it}. As we have shown, this is further equivalent to choosing a price contingent on the demand state z_{it}^D. That is, for each state realization z_{it}^D, the firm chooses a price p_{it} that maximizes the conditional expected value

$$p_{it}(z_{it}^D) = \arg\max_{p_{it}} \mathbb{E}_{it}[V(p_{it}, d(p_{it}, z_{it}^D), \mathbf{z}_{it}^V)|z_{it}^D] \tag{33}$$

where we have substituted firms' demand into the value function. We now consider a log-linear approximation around a deterministic steady state of this model, using hats to denote log deviations. The approximated policy function must evidently satisfy

$$\hat{p}_{it} = \tilde{\boldsymbol{\omega}}'_{1,it}\mathbb{E}_{it}\left[\hat{\mathbf{z}}_{it}^V|\hat{z}_{it}^D\right] \tag{34}$$

for some n_V-sized vector $\tilde{\boldsymbol{\omega}}_{1,it}$. Under the assumption that the shocks $\hat{\mathbf{z}}_{it}^V$ and $\hat{z}_{it}^D$ are normally distributed, optimal prices can further be written as

$$\hat{p}_{it} = \omega_{1,it}\hat{z}_{it}^D \tag{35}$$

for some scalar $\omega_{1,it}$. The coefficients $\omega_{1,it}$, the slopes of firms' supply functions in their demand states, then determine the motion for the economy's log-linearized ideal price index when averaged across firms, *i.e.*, $\hat{P}_t = \int_0^1 \hat{p}_{it}\, \mathrm{d}i$. This concludes the "firm block" of the model.

Following McKay and Wolf (2023), we assume that the aggregate dynamics of our economy can be summarized as

$$\mathcal{H}_x\mathbf{x} + \mathcal{H}_\varepsilon\boldsymbol{\varepsilon} = \mathbf{0} \tag{36}$$

where x_t is an n_x-dimensional vector of endogenous variables (such as the ideal price index $\hat{P}_t$), ε_t is an n_ε-dimensional vector of Gaussian structural shocks, and $\mathcal{H}_z$ and $\mathcal{H}_\varepsilon$ are conforming matrices.[12] Equation 36, for example, contains the relevant first-order conditions and market-clearing conditions that determine the dynamics of an economy. Of course, the matrices $\mathcal{H}_x$ and $\mathcal{H}_\varepsilon$ are dependent on firms' supply functions through the scalars $\omega_{1,it}$.

[12] Following McKay and Wolf (2023), we use boldface notation to stack the time paths for all variables (*e.g.* $\mathbf{x} = (\mathbf{x}'_1, \ldots, \mathbf{x}'_{n_x})'$). The matrices $\mathcal{H}_x$ and $\mathcal{H}_\varepsilon$ are conformable matrices that map bounded sequences to the space of bounded sequences.

Given $\omega_{1,it}$, we can solve for the equilibrium dynamics of the system summarized by Equation 36. Our additional "rational expectations" restriction then imposes that the value of $\omega_{1,it}$ is consistent with the equilibrium law of motion for prices given by Equation 34. As argued by McKay and Wolf, many of the parametric structural models commonly used for counterfactual analysis fit into the general framework of Equation 36. Our supply function approach simply asserts that the coefficients $\mathcal{H}_x$ and $\mathcal{H}_\varepsilon$ are consistent with the information underlying firm decision-making. We thus argue that supply functions can be embedded and studied within a general class of commonly used macroeconomic models. Validating the utility of this approach, Nikolakoudis (2024) incorporates supply functions along the input margin in a production network economy using these methods.

5 Quantification

In this final section, we quantify our model's implications for inflation-output tradeoffs. In particular, we compute the model's predictions for the slope of aggregate supply as a function of state-dependent uncertainty. In the United States, aggregate supply dramatically steepens during the 1970s and post-Covid crisis, consistent with empirical evidence (Ball and Mazumder, 2011; Hazell et al., 2022; Cerrato and Gitti, 2022). Across countries, relative uncertainty can help account for the vast and well-documented differences in the relationship between inflation and real outcomes (Lucas, 1973; Ball et al., 1988).

5.1 Methods

Our goal is to calculate the model-implied elasticity of aggregate supply and compare this to empirical evidence. To do this, we use Theorems 1 and 2 to write the slope of aggregate supply in terms of three parameters as well as *sufficient statistics* summarizing uncertainty about endogenous variables. That is,

$$\hat{\epsilon}_t^S = \gamma \frac{\kappa^M + \frac{\hat{\alpha}_{1,t}}{\gamma(1+\eta\hat{\alpha}_{1,t})}(1-\kappa^M)}{(1-\frac{\eta\hat{\alpha}_{1,t}}{1+\eta\hat{\alpha}_{1,t}})(1-\kappa^M)} \qquad \text{where } \hat{\alpha}_{1,t} = \frac{\eta\hat{\sigma}_{P,t}^2 + \hat{\sigma}_{\mathcal{M},\Psi,t} + \hat{\sigma}_{P,\Psi,t} + \eta\hat{\sigma}_{\mathcal{M},P,t}}{\hat{\sigma}_{\Psi,t}^2 - \eta\hat{\sigma}_{\mathcal{M},\Psi,t} + \eta\hat{\sigma}_{P,\Psi,t} - \eta^2\hat{\sigma}_{\mathcal{M},P,t}} \quad (37)$$

where the $\hat{\sigma}_{\cdot,t}$ denote period-by-period uncertainty about demand shifters Ψ, the price level P, and real marginal costs $\mathcal{M}$.

Our approach is to calibrate (γ, η, κ^M) and directly measure the uncertainties. Compared to the alternative of structurally estimating all of the model's deep parameters and the stochastic processes for all exogenous shocks, this approach has two main benefits. First, there is a relatively simple mapping from data to theory. In particular, our estimation of firms' supply function slopes is independent of the general equilibrium "block" of the economy and relies only on firms' beliefs about payoff-relevant variables. Second, we bypass the theoretically possible issue of multiple equilibria: our calculations are valid in the model regardless of whether measured uncertainty is due to variation in fundamentals or due to equilibrium selection. A limitation is that the method precludes us from studying counterfactuals in which, for example, firms' uncertainty endogenously responds to policy changes. We leave this kind of counterfactual analysis to future work. Nevertheless, in Appendix D.4, we outline a method for counterfactual analysis and provide further intuition for when the fixed point of Theorem 3 may feature multiple equilibria.

Structural Parameters. The period length controls the decision horizon of the firm. We set this to one quarter to be consistent with findings in US micro data that the duration of the median posted price is four months (Bils and Klenow, 2004). The elasticity of substitution between goods, η, controls the elasticity of demand faced by firms and, therefore, the relative importance of nominal and real shocks for the firm (see Corollary 1). We calibrate $\eta = 8$ to match the own-price elasticities of demand estimated by Hottman et al. (2016) using retail scanner data in the United States. The parameter γ controls the elasticity of real marginal costs to real output. We set $\gamma = 0.11$ to match the estimates of Gagliardone et al. (2023), who use micro-data from Belgian firms to estimate the elasticity of real marginal costs to output. The precision of firms' signals about the money supply, summarized by the signal-to-noise ratio κ^M, shifts the average slope of aggregate supply. We calibrate this to match an average aggregate supply slope of 0.11 as estimated by Hazell et al. (2022), yielding $\kappa^M = 0.29$. This allows us to isolate all time variation in the slope of aggregate supply as a consequence of time-varying uncertainty.

Uncertainty. The crucial remaining ingredient is firms' uncertainty about demand, aggregate prices, and real marginal costs. To our knowledge, there are no datasets that directly measure firms' multidimensional uncertainty about these objects over a long period of time. Therefore, we proxy for firms' subjective uncertainty using a simple statistical model. Specifically, we use quarterly data on real GDP, inflation (GDP deflator), and capacity-utilization-adjusted TFP from 1960 Q1 to 2024 Q4. Using the model's structure, we construct the aggregate component of real marginal costs as $\mathcal{M}_t = Y_t^{\gamma}/A_t$, where Y_t is real GDP, $\gamma = 0.11$ controls wealth effects in labor supply, and A_t is TFP. We model real GDP growth, inflation, and real marginal cost growth via a GARCH model (see Appendix D.1 for details). This gives us a quarterly measure of one-step-ahead uncertainty regarding macroeconomic variables. This corresponds to the uncertainty an economic agent would have about current-quarter macroeconomic variables if they observed past economic history and interpreted it using the statistical model. Our methods are also compatible with other measures of uncertainty, such as regime-specific covariances from a vector auto-regression (VAR) model.

To map our estimated uncertainties to Equation 37, we take two additional steps. First, we observe that real marginal cost uncertainty enters firms' supply-function choice only via the covariances of real marginal costs with demand and the price level (see Theorem 1). These covariances, in turn, depend only on uncertainty about the *aggregate* component of real marginal costs. This allows us to directly apply the GARCH estimates. Second, we observe that the firm-level demand shock is $\Psi_{it} = Y_t \vartheta_{it}$, where ϑ_{it} is an idiosyncratic demand shock. Since we lack direct estimates of firms' uncertainty about idiosyncratic demand shocks, we assume that idiosyncratic demand uncertainty is directly proportional to aggregate TFP uncertainty. We justify this based on the finding of Bloom et al. (2018) that the time-varying volatility of revenue-based TFP (TFPR) among manufacturing firms can be accurately modeled as directly proportional to time-varying volatility in aggregate conditions.[13] That is, we set $\hat{\sigma}^2_{\Psi,t} = \hat{\sigma}^2_{Y,t} + R^2 \hat{\sigma}^2_{\mathcal{M},t}$, where the $\hat{\sigma}_{\cdot,t}$ are the estimated GARCH variances and $R = 6.5$ from the quantitative estimates of Bloom et al. (2018).[14]

5.2 The Slope of Aggregate Supply in the US

Our estimates imply that the slope of aggregate supply has varied significantly in the US since 1960. Panel A of Figure 4 reports an annual time-series of our estimated slope, and Panel B reports average values over macroeconomic regimes. Aggregate supply significantly steepens in two episodes: the 1970s, coincident with the Oil Crisis, and the 2020s, coincident with the post-Covid inflation. Aggregate supply is flatter and roughly constant in the 1960s, the Great Moderation, the Great Recession, and the recovery from the Great Recession.

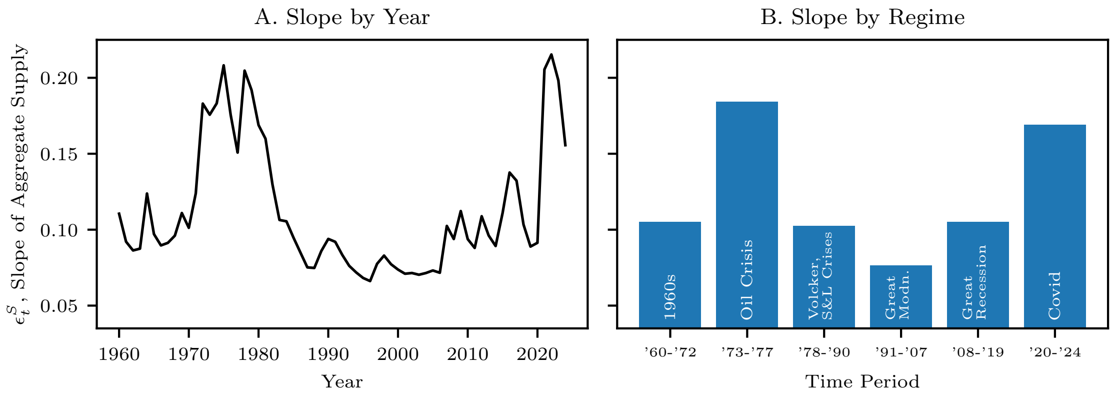
Comparison to Empirical Estimates. The slope of aggregate supply ϵ_t^S is defined as the relative response of the price level and real GDP to a monetary expansion in the same quarter. In other words, ϵ_t^S measures how inflationary are aggregate demand shocks in the short-run or the inverse "sacrifice ratio" for monetary policymakers. Therefore, the object of comparison in the data is the relative elasticity of inflation and real outcomes to an identified aggregate demand shock or, equivalently, the elasticity of inflation to real outcomes when the latter is instrumented with an identified aggregate demand shock.

A typical observation in research on US business cycles is that aggregate demand shocks in the data induce puzzlingly little short-run inflation: that is, in our language, short-run

[13] Foster et al. (2008) show that cross-firm variation in revenue total factor productivity (TFPR) derives almost exclusively from demand differences rather than marginal cost differences within specific industries, justifying our assumption that most "micro volatility" is demand volatility.

[14] Figure A1 in the Appendix plots each component of estimated uncertainty over time.

Figure 4: The Slope of Aggregate Supply in the United States Over Time



Notes: This figure shows the model implied slope of aggregate supply in the United States. The slope ϵ_t^S is defined by Equation 37. As described in Section 5.1, we set $\eta = 8$ and $\gamma = 0.11$ based on the literature, calibrate $\kappa^M = 0.29$ to match an average slope of 0.11 (Hazell et al., 2022), and estimate time-varying uncertainties from a GARCH model of quarterly-frequency GDP growth, inflation, and real marginal cost growth. Panel A displays the estimated slope averaged to the annual level. Panel B shows average values of the estimated slope in different macroeconomic regimes, defined on the horizontal axis and labeled inside the bars.

aggregate supply is relatively flat. For example, Ramey (2016) summarizes a large literature documenting relatively small responses of inflation to externally identified monetary policy shocks and Hazell et al. (2022) use panel data from US states to show that there is limited pass-through from unemployment to price inflation. Two particularly important observations in these studies of US economic history are the lack of deflation in response to the crash of aggregate demand in the Great Recession and the lack of inflation after the large subsequent monetary expansion (see also Coibion and Gorodnichenko, 2015; Bobeica and Jarociński, 2019). Our calculations are consistent with a low average slope of aggregate supply as well as a low slope during the Great Recession and recovery.

Nonetheless, studies that have estimated a *time-varying* slope of aggregate supply in the United States have found larger values in two periods, the 1970s and 2020s. Our model generates these dynamics as an endogenous consequence of abrupt changes in the composition of uncertainty. We make this comparison quantitatively in Table 1, which compares our predicted dynamics in the slope of aggregate supply to external estimates. The first panel is based on the study of Ball and Mazumder (2011), who estimate a time-varying relationship between inflation and the output gap.[15] Our model qualitatively matches a steepening of

[15]These authors construct the output gap using the Congressional Budget Office's estimate of potential real GDP. This corresponds to variation in real GDP induced by aggregate demand shocks if aggregate supply shocks move output together with potential output.

Table 1: Comparing the Model's Prediction to External Estimates

	Change in slope of aggregate supply relative to base period, compared to estimates from						
	A. Ball and Mazumder (2011)			B. Hazell et al. (2022)		C. Cerrato and Gitti (2022)	
	1960-1972	1973-1984	1985-2007	1978-1990	1991-2018	1991-2019	2021-2023
Data	—	+175%	-32%	—	-51%	—	+145%
Model	—	+58%	-25%	—	-28%	—	+112%

Notes: This table compares the model's estimates for the slope of aggregate supply with external empirical estimates. The values are percent changes in the slope of aggregate supply, relative to the base period (*i.e.*, the first period in each panel). The three panels correspond to comparisons with different studies: in Ball and Mazumder (2011), Column 4 of Table 3; in Hazell et al. (2022), Panel B, Columns 3 and 4 of Table II; in Cerrato and Gitti (2022), Column 2 of Table 2.

aggregate supply during the oil crisis and Volcker disinflation (1973-1984), while aggregate supply is relatively flat before and after. The second panel is based on the estimates of Hazell et al. (2022) who, as mentioned above, use state-level data in the United States to isolate demand-driven variation in real conditions. Our model can account for about 1/2 of the authors' estimated flattening of the Phillips curve from 1978-1990 to 1991-2018.

The third panel shows the consistency of our results with the behavior of aggregate supply before and after the Covid crisis based on the estimates of Cerrato and Gitti (2022), who use data from US metropolitan statistical areas (MSAs) to isolate demand shocks. Quantitatively, our model accounts for about 4/5 of the steepening of aggregate supply between the pre-Covid and post-Covid periods. Both the model predictions and empirical evidence imply that aggregate demand shocks were relatively inflationary during this period—consistent with both a large inflationary effect (and small real effect) of fiscal expansion and a large disinflationary effect (and small real effect) of monetary tightening.

These results suggest that our model's qualitative predictions for variation in the slope of aggregate supply are consistent with US empirical evidence. Of course, our baseline model is lacking several features required to make the model fully consistent with price stickiness at the microeconomic level and realistically sluggish inflation dynamics at the macroeconomic level. Theoretically, we have shown that such features can be integrated into the supply-function approach, while maintaining the essential properties of the latter (Section 2.3).[16] We leave a more complete quantitative exploration in a richer model to further research.

[16] The extension to sticky prices (Appendix B.4) would assist on both fronts. Another possibility, which is conceptually straightforward to integrate into the model, is to assume that firms do not observe the history of macroeconomic aggregates and therefore form beliefs using a latent-state (Kalman filtering) model. As shown by Woodford (2003a) in a model of price-setting, the combination of incomplete information with strategic interaction can generate realistically slow responses of inflation to shocks.

Rising Markups and Flattening Supply. While our analysis thus far has focused on time variation in only uncertainty, in principle time variation in other structural parameters could affect the slope of aggregate supply in our model. In Appendix D.2, we show that a version of our quantification that feeds in a secular decline of the elasticity of substitution, or a secular increase in average markups, predicts an even more pronounced flattening of the aggregate supply curve over time (Figure A3). We leave a fuller empirical investigation of the model's joint predictions for aggregate supply and markups to future work.

5.3 The Slope of Aggregate Supply Around the World

A cursory glance at Figure 4 suggests that aggregate supply is relatively steep in times during "inflationary crises" in the US and relatively flat otherwise. Our quarterly time-series for ϵ_t^S has a correlation of 0.93 with one-quarter-ahead *uncertainty* regarding inflation and a correlation of 0.62 with the quarterly *level* of inflation (Figure A2). The deviation between the level of inflation and inflation uncertainty primarily occurs near "turning points" of the inflation time series: for example, uncertainty is high despite relatively tame inflation in 2021Q1, on the heels of the Covid lockdown, while uncertainty is only moderate despite very high inflation in 1981Q2, near the peak of the Volcker tightening cycle. Nonetheless, the high correlation between these variables presents a challenge for differentiating the *qualitative* predictions of our model from alternatives in which price rigidity depends on the baseline rate of inflation or a one-dimensional summary of inflation volatility (e.g., Ball et al., 1988). These include models of endogenous price reset probabilities and models of menu costs.

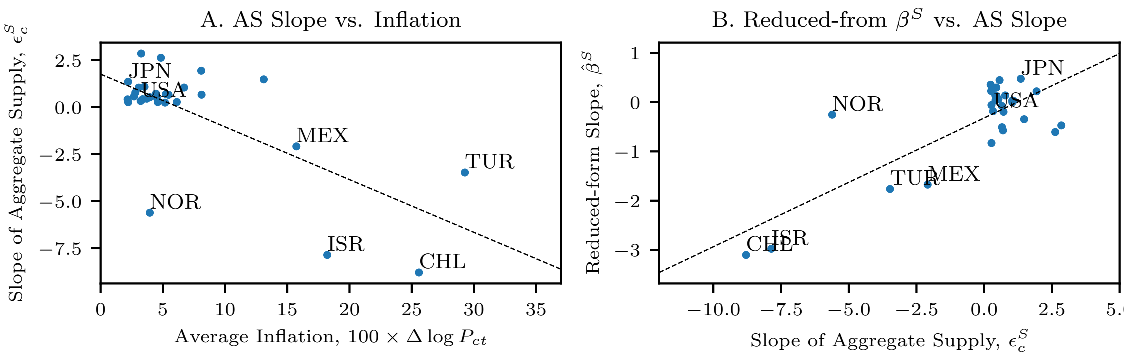
To help distinguish between the roles of the level of inflation and uncertainty about inflation, we conduct a cross-country analysis in the spirit of Lucas (1973) and Ball et al. (1988). We compile annual data on real GDP growth, GDP deflator inflation, and TFP growth from 1960-2019 for countries in the OECD.[17] In each country, we model these variables as a one-lag VAR in first differences. Using the same structural parameters and mapping to the model described above, we construct country-level measures of uncertainty regarding demand, inflation, and real marginal costs and a model-implied slope of aggregate supply.[18]

Our first finding is that our cross-country estimates of the slope of aggregate supply are *not* positively predicted by average inflation (Panel A of Figure 5). This is in sharp

[17]We construct TFP as a Solow residual with elasticities 1/3 and 2/3 on the real capital stock and total labor hours, respectively. We drop three outliers from our calculation, Greece, Iceland, and Sweden, for which we calculate a slope of aggregate supply and/or inflation-output relationship more than 3 standard deviations away from the median.

[18]While our calculation embodies numerous assumptions, we argue that the main ingredients capture intuitive differences across countries. For example, uncertainty about inflation is the lowest among OECD countries in the United States, Germany, and Canada, and the highest in Chile, Israel, and Mexico.

Figure 5: International Evidence on the Slope of Aggregate Supply



Notes: This figure summarizes our international estimates of the slope of aggregate supply. In each panel, an observation is an OECD country and the country-level slopes of aggregate supply are calculated using the methods described in Section 5.3. Panel A plots the relationship of the model-implied slopes versus mean levels of inflation (GDP deflator) from 1960-2019. Panel B plots the relationship between the model-implied slope of aggregate supply and the "reduced form" slope estimate, $\hat{\beta}_c^S = \text{Cov}[\Delta \log Y_{ct}, \Delta \log P_{ct}]/\text{Var}[\Delta \log Y_{ct}]$. Dashed lines are from linear regressions and the labels identify selected countries by three-letter codes.

contrast to our within-country estimates for the United States. Internationally, the slope of aggregate supply is predicted to be *negative* for the countries with the highest rate of inflation—a consequence of the fact that the same countries have very highly correlated uncertainty about the price level and real marginal costs.

We next show that our model's predictions for cross-country relationships between inflation and output growth line up with the data. Panel B of Figure 5 shows a positive relationship between our predicted slope and the year-on-year regression coefficient of inflation on real output growth: for each country c, $\hat{\beta}_c^S = \frac{\text{Cov}[\Delta \log Y_{ct}, \Delta \log P_{ct}]}{\text{Var}[\Delta \log Y_{ct}]}$. Thus, in the raw data, output growth is more inflationary in countries for which we predict a steep slope of aggregate supply. Of course, the classic conceptual issue with interpreting inflation-output "correlations" as the slope of aggregate supply is that macroeconomic outcomes are determined by both demand and supply shocks. In Appendix D.3, we construct a model-derived instrument which isolates exogenous variation in the money supply. Our IV estimate of the slope of aggregate supply is also positively correlated with our predicted slope.

While these findings on a small set of countries are merely suggestive, they imply that our model and its emphasis on *relative* uncertainties might help account for the enormous heterogeneity in inflation-output dynamics across countries. Moreover, the model has predictive power over and above models in which the slope of aggregate supply depends positively on only the mean or volatility of inflation (e.g., Ball et al., 1988).[19]

[19] In Table A2, we more formally demonstrate this in a cross-country regression analysis.

6 Conclusion

In this paper, we enrich firms' supply decisions by allowing them to choose supply functions that describe the price charged at each quantity of production. We show how to model supply functions in a macroeconomic setting and characterize how the optimal supply function depends on the elasticity of demand and the nature of uncertainty that firms face. Our framework yields rich implications when embedded in an otherwise standard monetary business cycle model. We find that a higher elasticity of demand and increased uncertainty about the price level relative to demand endogenously steepen aggregate supply. When mapped to the data, our model generates variation in the slope of aggregate supply that is consistent with empirical evidence within the US and across countries.

On the basis of our analysis, we argue that supply schedules warrant serious consideration as an alternative model of firm conduct in macroeconomics for three core reasons. First, most existing work assumes that firms set a price in advance and commit to supply at the market-clearing quantity. Our results emphasize that this is not generally an optimal way for a firm to behave and that the macroeconomic conclusions that one draws about the effects of uncertainty, the propagation of monetary and productivity shocks, and the role of market power are highly sensitive to this choice. For example, the price-setting assumption maximizes the degree of monetary non-neutrality and leaves no role for market power. Second, we have shown that working with supply schedules is analytically tractable under the standard assumptions in the literature and can be done in a large class of linearized macroeconomic models of the kind studied by, for example, McKay and Wolf (2023). Finally, taking the supply-schedule perspective yields economic predictions that are consistent with broad trends in US aggregate supply over the last 60 years and cross-sectional patterns in aggregate supply around the world.

Within the context of supply schedules and the macroeconomy, our study is only a first exploration; there remains much to examine. Recent work has expanded upon our analysis of the macroeconomic implications of supply function equilibrium to consider learning from input markets. Of particular note, Hellwig and Venkateswaran (2024) study a dispersed information model in which firms nonetheless have enough information to implement their full-information pricing policy. Their analysis also shows how our model of supply function competition in general equilibrium can be fruitfully integrated into other dynamic macroeconomic models. Moreover, Nikolakoudis (2024) considers a framework in which firms learn from input prices in a production network. Further exploration of how different market structures affect firms' supply decisions appears a promising avenue for future research.

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Appendices

A Omitted Proofs

A.1 Proof of Theorem 1

Proof. Fix a supply function f. The realized price of the firm in state z solves $f(\hat{p}(z), z\hat{p}(z)^{-\eta}) = 0$. As we placed no restrictions on f, it is equivalent to think of the firm as choosing $\hat{p}$ directly. For a given choice of $\hat{p}$, the firm's payoff is given by:

$$J(\hat{p}) = \int_{\mathbb{R}^4_{++}} \Lambda \left(\frac{\hat{p}(z)}{P} - \mathcal{M} \right) z\hat{p}(z)^{-\eta} \, \mathrm{d}G\left(\Lambda, P, \mathcal{M}, z\right) \tag{38}$$

where G is the cumulative distribution function representing the firm's beliefs. We therefore study the problem:

$$\sup_{\hat{p}:\mathbb{R}_+ \to \mathbb{R}_{++}} J(\hat{p}) \tag{39}$$

Given a solution $\hat{p}$ for how firms optimally adapt their prices to demand, we will recover the optimal plan f for how firms optimally set a supply function.

We first derive Equation 8 using variational methods. Consider a variation $\tilde{p}(z) = p(z) + \varepsilon h(z)$. The expected payoff under this variation is:

$$J(\varepsilon; h) = \int_{\mathbb{R}^4_{++}} \Lambda \left(\frac{p(z) + \varepsilon h(z)}{P} - \mathcal{M} \right) z \left(p(z) + \varepsilon h(z)\right)^{-\eta} \mathrm{d}G\left(\Lambda, P, \mathcal{M}, z\right) \tag{40}$$

A necessary condition for the optimality of a function p is that $J_\varepsilon(0; h) = 0$ for all $F-$measurable h. Taking this derivative and setting $\varepsilon = 0$, we obtain:

$$0 = \int_{\mathbb{R}^4_{++}} \left[\Lambda \frac{h(z)}{P} zp(z)^{-\eta} - \eta \Lambda h(z) \left(\frac{p(z)}{P} - \mathcal{M} \right) zp(z)^{-\eta-1} \right] \mathrm{d}G\left(\Lambda, P, \mathcal{M}, z\right) \tag{41}$$

Consider h functions given by the Dirac delta functions on each z, $h(z) = \delta_z$. This condition becomes:

$$0 = \int_{\mathbb{R}^3_{++}} \left[\Lambda \frac{1}{P} tp(t)^{-\eta} - \eta \Lambda \left(\frac{p(t)}{P} - \mathcal{M} \right) tp(t)^{-\eta-1} \right] g(\Lambda, P, \mathcal{M}, t) \, \mathrm{d}\Lambda \, \mathrm{d}P \, \mathrm{d}\mathcal{M} \tag{42}$$

for all $t \in \mathbb{R}_{++}$. This is equivalent to:

$$\begin{aligned} 0 &= \int_{\mathbb{R}^3_{++}} \left[\Lambda \frac{1}{P} t p(t)^{-\eta} - \eta \Lambda \left(\frac{p(t)}{P} - \mathcal{M} \right) t p(t)^{-\eta-1} \right] g(\Lambda, P, \mathcal{M} | t) \, \mathrm{d}\Lambda \, \mathrm{d}P \, \mathrm{d}\mathcal{M} \\ &= (1-\eta) \mathbb{E}\left[\Lambda \frac{1}{P} | z = t \right] t p(t)^{-\eta} + \eta \mathbb{E}\left[\Lambda \mathcal{M} | z = t \right] t p(t)^{-\eta-1} \end{aligned} \tag{43}$$

Thus, we have that an optimal solution necessarily follows:

$$p(t) = \frac{\eta}{\eta - 1} \frac{\mathbb{E}[\Lambda \mathcal{M} | z = t]}{\mathbb{E}[\Lambda P^{-1} | z = t]} \tag{44}$$

as claimed in Equation 8.

We now evaluate the expectations. Using log-normality,

$$\begin{aligned} \mathbb{E}[\Lambda \mathcal{M} | z = t] &= \exp\left\{ \mu_{\Lambda|z}(t) + \mu_{\mathcal{M}|z}(t) + \frac{1}{2}\sigma^2_{\Lambda|z} + \frac{1}{2}\sigma^2_{\mathcal{M}|z} + \sigma_{\Lambda,\mathcal{M}|z} \right\} \\ \mathbb{E}[\Lambda P^{-1} | z = t] &= \exp\left\{ \mu_{\Lambda|z}(t) - \mu_{P|z}(t) + \frac{1}{2}\sigma^2_{\Lambda|z} + \frac{1}{2}\sigma^2_{P|z} - \sigma_{\Lambda,P|z} \right\} \end{aligned} \tag{45}$$

where $\mu_{X|z} = \mathbb{E}[\log X \,|\, \log z]$ and $\sigma_{X,Y|z} = \mathrm{Cov}[\log X, \log Y \,|\, \log z]$. Thus,

$$\frac{\mathbb{E}[\Lambda \mathcal{M} | z = t]}{\mathbb{E}[\Lambda P^{-1} | z = t]} = \exp\left\{ \mu_{\mathcal{M}|z}(t) + \mu_{P|z}(t) + \frac{1}{2}\sigma^2_{\mathcal{M}|z} - \frac{1}{2}\sigma^2_{P|z} + \sigma_{\Lambda,\mathcal{M}|z} + \sigma_{\Lambda,P|z} \right\} \tag{46}$$

Using standard formulas for Gaussian conditional expectations,

$$\begin{aligned} \mu_{\mathcal{M}|z}(t) &= \mu_{\mathcal{M}} + \frac{\sigma_{\mathcal{M},z}}{\sigma^2_z}(\log t - \mu_z) & \mu_{P|z}(t) &= \mu_P + \frac{\sigma_{P,z}}{\sigma^2_z}(\log t - \mu_z) \\ \sigma^2_{\mathcal{M}|z} &= \sigma^2_{\mathcal{M}} - \frac{\sigma^2_{\mathcal{M},z}}{\sigma^2_z} & \sigma^2_{P|z} &= \sigma^2_P - \frac{\sigma^2_{P,z}}{\sigma^2_z} \\ \sigma_{\Lambda,\mathcal{M}|z} &= \sigma_{\Lambda,\mathcal{M}} - \frac{\sigma_{\Lambda,z}\sigma_{\mathcal{M},z}}{\sigma^2_z} & \sigma_{\Lambda,P|z} &= \sigma_{\Lambda,P} - \frac{\sigma_{\Lambda,z}\sigma_{P,z}}{\sigma^2_z} \end{aligned} \tag{47}$$

where:

$$\begin{aligned} \sigma^2_z &= \sigma^2_\Psi + \eta^2\sigma^2_P + 2\eta\sigma_{\Psi,P} & \sigma_{P,z} &= \sigma_{P,\Psi} + \eta\sigma^2_P \\ \sigma_{\mathcal{M},z} &= \sigma_{\mathcal{M},\Psi} + \eta\sigma_{\mathcal{M},P} & \sigma_{\Lambda,z} &= \sigma_{\Lambda,\Psi} + \eta\sigma_{\Lambda,P} \end{aligned} \tag{48}$$

We now combine these expressions with Equation 44 to derive the optimal supply function. We first observe that

$$\log p = \omega_0 + \omega_1 \log t \tag{49}$$

where:

$$\omega_0 = \log \frac{\eta}{\eta - 1} + \mu_{\mathcal{M}} + \mu_P - \omega_1 \mu_z + \frac{1}{2}\sigma^2_{\mathcal{M}|z} - \frac{1}{2}\sigma^2_{P|z} + \sigma_{\Lambda,\mathcal{M}|z} + \sigma_{\Lambda,P|z} \tag{50}$$

$$\omega_1 = \frac{\sigma_{\mathcal{M},z} + \sigma_{P,z}}{\sigma^2_z} = \frac{\sigma_{\mathcal{M},\Psi} + \eta\sigma_{\mathcal{M},P} + \sigma_{P,\Psi} + \eta\sigma^2_P}{\sigma^2_\Psi + \eta^2\sigma^2_P + 2\eta\sigma_{\Psi,P}} \tag{51}$$

Next, using the demand curve, we observe that $z = qp^\eta$. Therefore, $\log t = \log q - \eta \log p$. Substituting this into Equation 49, and re-arranging, we obtain

$$\log p = \alpha_0 + \alpha_1 \log q \tag{52}$$

where:

$$\alpha_0 = \frac{\omega_0}{1 - \eta\omega_1}, \qquad \alpha_1 = \frac{\omega_1}{1 - \eta\omega_1} \tag{53}$$

We finally derive the claimed expression for α_1,

$$\begin{aligned} \alpha_1 &= \frac{\frac{\sigma_{\mathcal{M},\Psi} + \eta\sigma_{\mathcal{M},P} + \sigma_{P,\Psi} + \eta\sigma^2_P}{\sigma^2_\Psi + \eta^2\sigma^2_P + 2\eta\sigma_{\Psi,P}}}{1 - \eta\frac{\sigma_{\mathcal{M},\Psi} + \eta\sigma_{\mathcal{M},P} + \sigma_{P,\Psi} + \eta\sigma^2_P}{\sigma^2_\Psi + \eta^2\sigma^2_P + 2\eta\sigma_{\Psi,P}}} \\ &= \frac{\sigma_{\mathcal{M},\Psi} + \eta\sigma_{\mathcal{M},P} + \sigma_{P,\Psi} + \eta\sigma^2_P}{\sigma^2_\Psi + \eta\sigma_{\Psi,P} - \eta\sigma_{\mathcal{M},\Psi} - \eta^2\sigma_{\mathcal{M},P}} \end{aligned} \tag{54}$$

Completing the proof.

□

A.2 Proof of Corollary 1

Proof. If $2\eta\sigma_{\mathcal{M},P} + \sigma_{\mathcal{M},\Psi} \geq \sigma_{P,\Psi}$, then the denominator of Equation 6 is decreasing in η. Moreover, if $\sigma_{\mathcal{M},P} \geq 0$, the numerator is increasing in η. Hence, α_1 is increasing in η whenever $\alpha_1 > 0$. □

A.3 Proof of Proposition 1

Proof. From the household's choice among varieties, the demand curve for each variety i is

$$\frac{p_{it}}{P_t} = \left(\frac{c_{it}}{\vartheta_{it} C_t}\right)^{-\frac{1}{\eta}} \tag{55}$$

From the intratemporal Euler equation for consumption demand *vs.* labor supply, the household equates the marginal benefit of supplying additional labor $w_{it} C_t^{-\gamma} P_t^{-1}$ with its marginal cost ϕ_{it}. Thus, variety-specific wages are given by

$$w_{it} = \phi_{it} P_t C_t^{\gamma} \tag{56}$$

From the intertemporal Euler equation between consumption and money today, the cost of holding an additional dollar today equals the benefit of holding an additional dollar today plus the value of an additional dollar tomorrow:

$$C_t^{-\gamma} \frac{1}{P_t} = \frac{1}{M_t} + \beta \mathbb{E}_t \left[C_{t+1}^{-\gamma} \frac{1}{P_{t+1}} \right] \tag{57}$$

Further, from the intertemporal choice between bonds, the cost of saving an additional dollar today equals the nominal interest rate $1+i_t$ times the value of an additional dollar tomorrow:

$$C_t^{-\gamma} \frac{1}{P_t} = \beta(1+i_t) \mathbb{E}_t \left[C_{t+1}^{-\gamma} \frac{1}{P_{t+1}} \right] \tag{58}$$

From Equations 57 and 58, we obtain:

$$\frac{1}{M_t} + \beta \mathbb{E}_t \left[C_{t+1}^{-\gamma} \frac{1}{P_{t+1}} \right] = \beta(1+i_t) \mathbb{E}_t \left[C_{t+1}^{-\gamma} \frac{1}{P_{t+1}} \right] \tag{59}$$

It follows that:

$$\frac{1}{M_t} = \beta i_t \mathbb{E}_t \left[C_{t+1}^{-\gamma} \frac{1}{P_{t+1}} \right] = \frac{i_t}{1+i_t} C_t^{-\gamma} \frac{1}{P_t} \tag{60}$$

where the second equality uses Equation 58 once again. This rearranges to:

$$C_t = \left(\frac{i_t}{1+i_t} \right)^{\frac{1}{\gamma}} \left(\frac{M_t}{P_t} \right)^{\frac{1}{\gamma}} \tag{61}$$

We next derive the interest rate. Substituting equation 61 into Equation 58, we obtain:

$$\frac{1+i_t}{i_t} \frac{1}{M_t} = \beta(1+i_t) \mathbb{E}_t \left[\frac{1+i_{t+1}}{i_{t+1}} \frac{1}{M_{t+1}} \right] \tag{62}$$

Dividing both sides by $(1+i_t)$, multiplying by M_t, and then adding one, we obtain:

$$\frac{1+i_t}{i_t} = 1 + \beta \mathbb{E}_t \left[\frac{1+i_{t+1}}{i_{t+1}} \frac{M_t}{M_{t+1}} \right] = 1 + \beta \mathbb{E}_t \left[\exp\{-\mu_M - \sigma_{t+1}^M \varepsilon_{t+1}^M\} \frac{1+i_{t+1}}{i_{t+1}} \right] \tag{63}$$

where the second equality exploits the fact that M_t follows a random walk with drift. If we guess that i_t is deterministic and define $x_t = \frac{1+i_t}{i_t}$, then we obtain that:

$$x_t = 1 + \delta_t x_{t+1} \tag{64}$$

where:

$$\delta_t = \beta \exp\left\{-\mu_M + \frac{1}{2}(\sigma_{t+1}^M)^2\right\} \tag{65}$$

We observe that $\delta_t \in [0, \beta]$ for all t due to the assumption that $\frac{1}{2}(\sigma_t^M)^2 \leq \mu_M$. Solving this equation forward, we obtain that for $T \geq 2$:

$$x_t = 1 + \delta_t \left(1 + \sum_{i=1}^{T-1} \prod_{j=1}^{i} \delta_{t+j}\right) + \delta_t \left(\prod_{j=1}^{T} \delta_{t+j}\right) x_{t+T+1} \tag{66}$$

Taking the limit $T \to \infty$, this becomes:

$$x_t = 1 + \delta_t \left(1 + \sum_{i=1}^{\infty} \prod_{j=1}^{i} \delta_{t+j}\right) + \delta_t \lim_{T\to\infty} \left(\prod_{j=1}^{T} \delta_{t+j}\right) x_{t+T+1} \tag{67}$$

where the final term can be bounded using the fact that $\delta_t \in [0, \beta]$:

$$0 \leq \delta_t \lim_{T\to\infty} \left(\prod_{j=1}^{T} \delta_{t+j}\right) x_{t+T+1} \leq \lim_{T\to\infty} \beta^{T+1} x_{t+T+1} \tag{68}$$

The household's transversality condition ensures that this upper bound is zero. Formally, the transversality condition (necessary for the optimality of the household's choices) is that:

$$\lim_{T\to\infty} \beta^T \frac{C_T^{-\gamma}}{P_T}(M_T + (1 + i_T)B_T) = 0 \tag{69}$$

Moreover, as $B_t = 0$ for all $t \in \mathbb{N}$, this reduces to $\lim_{T\to\infty} \beta^T \frac{C_T^{-\gamma}}{P_T} M_T = 0$. By Equation 60, we have that $\frac{x_t}{M_t} = \frac{C_t^{-\gamma}}{P_t}$. Thus, the transversality condition reduces to $\lim_{T\to\infty} \beta^T x_T = 0$. Combining this with Equation 68, we have that $\lim_{T\to\infty} \left(\prod_{j=1}^{T} \delta_{t+j}\right) x_{t+T+1} = 0$. An explicit formula for the interest rate follows:

$$\frac{1+i_t}{i_t} = 1 + \beta \exp\left\{-\mu_M + \frac{1}{2}(\sigma_{t+1}^M)^2\right\} \left(1 + \sum_{i=1}^{\infty} \prod_{j=1}^{i} \beta \exp\left\{-\mu_M + \frac{1}{2}(\sigma_{t+j+1}^M)^2\right\}\right) \tag{70}$$

The formulae in Equation 20 then follow. In particular, $\Psi_{it} = \vartheta_{it} C_t$ follows from comparing Equations 2 and 55. $P_t = \frac{i_t}{1+i_t} C_t^{-\gamma} M_t$ follows from Equation 61. $\Lambda_t = C_t^{-\gamma}$ is the households marginal utility from consumption. Finally, $\mathcal{M}_{it} = \frac{1}{z_{it}A_t}\frac{w_{it}}{P_t} = \frac{\phi_{it}C_t^{\gamma}}{z_{it}A_t}$ follows from Equation 56. □

A.4 Proof of Theorem 2

Proof. We begin by characterizing log-linear equilibria, which is achieved by the following Lemma:

Lemma 1 (Macroeconomic Dynamics with Supply Functions)**.** *If all firms use log-linear supply functions of the form in Equation 21, output in the unique log-linear temporary equilibrium follows:*

$$\log C_t = \tilde{\chi}_{0,t} + \frac{1}{\gamma}\frac{\kappa_t^A}{1-\omega_{1,t}\left(\eta - \frac{1}{\gamma}\right)(1-\kappa_t^A)}\log A_t + \frac{1}{\gamma}\frac{(1-\kappa_t^M)(1-\eta\omega_{1,t})}{1-\omega_{1,t}\left(\eta-\frac{1}{\gamma}\right)(1-\kappa_t^M)}\log M_t \quad (71)$$

and the aggregate price in the unique log-linear temporary equilibrium is given by:

$$\log P_t = \chi_{0,t} - \frac{\kappa_t^A}{1-\omega_{1,t}\left(\eta-\frac{1}{\gamma}\right)(1-\kappa_t^A)}\log A_t + \frac{\kappa_t^M + \frac{\omega_{1,t}}{\gamma}(1-\kappa_t^M)}{1-\omega_{1,t}\left(\eta-\frac{1}{\gamma}\right)(1-\kappa_t^M)}\log M_t \quad (72)$$

where $\chi_{0,t}$ and $\tilde{\chi}_{0,t}$ are constants that depend only on parameters (including $\alpha_{1,t}$) and past shocks to the economy.

Proof. We suppress dependence on t for ease of notation. Consider a plan:

$$\log p_i = \log\tilde{\alpha}_{0,i} + \alpha_1 \log q_i \quad (73)$$

where $\tilde{\alpha}_{0,i} = e^{\alpha_{0,i}}$. The demand-supply relationship that the firm faces is:

$$\log p_i = -\frac{1}{\eta}(\log q_i - \log\Psi) + \log P \quad (74)$$

The realized quantity therefore is:

$$\log q_i = \frac{-\eta}{1+\eta\alpha_1}\log\tilde{\alpha}_{0,i} + \frac{1}{1+\eta\alpha_1}\log\Psi_i P^{\eta} \quad (75)$$

and the realized price is:

$$\log p_i = \frac{1}{1+\eta\alpha_1}\log\tilde{\alpha}_{0,i} + \frac{\alpha_1}{1+\eta\alpha_1}\log\Psi_i P^{\eta} \quad (76)$$

It is useful to make the change of variables $\omega_1 = \frac{\alpha_1}{1+\eta\alpha_1}$, which implies that we may write

$$\log p_i = (1 - \eta\omega_1) \log \tilde{\alpha}_{0,i} + \omega_1 \log \Psi_i P^\eta \tag{77}$$

Our goal is to express dynamics only as a function of ω_1. We first find the optimal $\alpha_{0,i}$ in terms of ω_1. The firm therefore solves:

$$\max_{\tilde{\alpha}_{0,i}} \mathbb{E}_i \left[\Lambda \left(\frac{p_i}{P} - \mathcal{M}_i \right) \left(\frac{p_i}{P} \right)^{-\eta} \Psi_i \right] \tag{78}$$

Substituting for the realized price using the demand-supply relationship yields:

$$\max_{\tilde{\alpha}_{0,i}} \mathbb{E} \left[\Lambda \left(\frac{\tilde{\alpha}_{0,i}^{1-\eta\omega_1}}{P} (\Psi_i P^\eta)^{\omega_1} - \mathcal{M}_i \right) \tilde{\alpha}_{0,i}^{\eta^2\omega_1 - \eta} (\Psi_i P^\eta)^{1-\eta\omega_1} \right] \tag{79}$$

The optimal $\tilde{\alpha}_{0,i}$ is:

$$\tilde{\alpha}_{0,i}^{1-\eta\omega_1} = \frac{\eta}{\eta - 1} \frac{\mathbb{E}_i[\Lambda \mathcal{M}_i (\Psi_i P^\eta)^{1-\eta\omega_1}]}{\mathbb{E}_i[\frac{\Lambda}{P} (\Psi_i P^\eta)^{1-\eta\omega_1+\omega_1}]} \tag{80}$$

Substituting back into the realized price yields:

$$p_i = \frac{\eta}{\eta - 1} \frac{\mathbb{E}_i[\Lambda \mathcal{M}_i (\Psi_i P^\eta)^{1-\eta\omega_1}]}{\mathbb{E}_i[\frac{\Lambda}{P} (\Psi_i P^\eta)^{1-\eta\omega_1+\omega_1}]} (\Psi_i P^\eta)^{\omega_1} \tag{81}$$

We may express this only in terms of P by using Proposition 1, where we let $I = \frac{1+i}{i}$ for ease of notation:

$$\begin{aligned} p_i = {} & \frac{\eta}{\eta - 1} \frac{\mathbb{E}_i \left[\phi(z_i A)^{-1} \left(\vartheta_i I^{-\frac{1}{\gamma}} P^{-\frac{1}{\gamma}} M^{\frac{1}{\gamma}} P^\eta \right)^{1-\eta\omega_1} \right]}{\mathbb{E}_i \left[I^{1-\frac{1}{\gamma}(1+\omega_1-\eta\omega_1)} M^{\frac{1}{\gamma}(1+\omega_1-\eta\omega_1)-1} \vartheta^{1+\omega_1-\eta\omega_1} P^{\left(\eta-\frac{1}{\gamma}\right)(1+\omega_1-\eta\omega_1)} \right]} \\ & \times \left(\vartheta_i I^{-\frac{1}{\gamma}} M^{\frac{1}{\gamma}} P^{\eta-\frac{1}{\gamma}} \right)^{\omega_1} \end{aligned} \tag{82}$$

Given the ideal price index formula (Equation 14), P must satisfy the aggregation:

$$P^{1-\eta} = \mathbb{E} \left[\vartheta_i p_i^{1-\eta} \right] \tag{83}$$

where the expectation is over the cross-section of firms. We guess and verify that the aggregate price is log-linear in aggregates

$$\log P = \chi_0 + \chi_A \log A + \chi_M \log M \tag{84}$$

Moreover, if the p_i are log-normally distributed (we will verify this below), then:

$$\log P = \mathbb{E}[\log p_i] + \frac{1}{2(1-\eta)}\text{Var}((1-\eta)\log p_i) + \text{const} \tag{85}$$

We first simplify the numerator of the first term by collecting all the terms involving s_i^A and s_i^M:

$$\begin{aligned}\log \mathbb{E}_i\left[\phi_i(z_i A)^{-1}\left(\vartheta I^{-\frac{1}{\gamma}} P^{-\frac{1}{\gamma}} M^{\frac{1}{\gamma}} P^{\eta}\right)^{1-\eta\omega_1}\right] &= \left[-\kappa^A + \kappa^A\left(\eta - \frac{1}{\gamma}\right)\chi_A(1-\eta\omega_1)\right] s_i^A \\ + \left[\chi_M\left(\eta - \frac{1}{\gamma}\right)(1-\eta\omega_1)\kappa^M + \frac{1}{\gamma}(1-\eta\omega_1)\kappa^M\right] s_i^M + \text{const}\end{aligned} \tag{86}$$

where the constants are independent of signals. We similarly simplify the denominator of the second term:

$$\begin{aligned}&\log \mathbb{E}_i\left[I^{1-\frac{1}{\gamma}(1+\omega_1-\eta\omega_1)} M^{\frac{1}{\gamma}(1+\omega_1-\eta\omega_1)-1} \vartheta^{1+\omega_1-\eta\omega_1} P^{\left(\eta-\frac{1}{\gamma}\right)(1+\omega_1-\eta\omega_1)}\right] = \\ &\left[\chi_A\left(\eta - \frac{1}{\gamma}\right)(1+\omega_1-\eta\omega_1)\kappa^A\right] s_i^A \\ &+ \left[\left[\frac{1}{\gamma}(1+\omega_1-\eta\omega_1) - 1\right]\left(\kappa^M\right) + \chi_M\left(\eta - \frac{1}{\gamma}\right)(1+\omega_1-\eta\omega_1)\left(\kappa^M\right)\right] s_i^M \\ &+ \text{const}\end{aligned} \tag{87}$$

where the constants are again independent of signals. Finally, we can simplify the last term:

$$\log\left(\vartheta_i I^{-\frac{1}{\gamma}} M^{\frac{1}{\gamma}} P^{\eta-\frac{1}{\gamma}}\right)^{\omega_1} = \omega_1 \chi_A\left(\eta - \frac{1}{\gamma}\right)\log A + \omega_1\left[\chi_M\left(\eta - \frac{1}{\gamma}\right) + \frac{1}{\gamma}\right]\log M + \text{const} \tag{88}$$

where the constants are independent of the aggregate shocks. Hence, $\log p_i$ is indeed normally distributed and its variance is independent of the realization of aggregate shocks. We can now collect terms to verify our log-linear guess. Substituting the resulting expression for $\log p_i$ and our guess for $\log P$ from Equation 84 into Equation 85, and solving for χ_A by collecting coefficients on $\log A$ yields:

$$\chi_A = -\frac{\kappa^A}{1-\omega_1\left(\eta - \frac{1}{\gamma}\right)(1-\kappa^A)} \tag{89}$$

We may similarly solve for χ_M:

$$\chi_M = \frac{\kappa^M + \frac{\omega_1}{\gamma}(1-\kappa^M)}{1-\omega_1\left(\eta - \frac{1}{\gamma}\right)(1-\kappa^M)} \tag{90}$$

This proves the dynamics for the price level. The dynamics for consumption then follow from Proposition 1. □

With this characterization in hand, by Equation 71 and market clearing $C_t = Y_t$, we have:

$$\log M_t = \frac{1}{\tilde{\chi}_{M,t}} \left(\log Y_t - \tilde{\chi}_{A,t} \log A_t - \tilde{\chi}_{0,t} \right) \tag{91}$$

Substituting for $\log M_t$ in Equation 72 and defining $\log \bar{P}_t = \chi_{0,t} - \epsilon_t^S \tilde{\chi}_{0,t}$ and $\delta_t = \chi_{A,t} - \epsilon_t^S \tilde{\chi}_{A,t}$ then yields Equation AS:

$$\log P_t = \log \bar{P}_t + \epsilon_t^S \log Y_t + \delta_t \log A_t \tag{92}$$

Doing a similar substitution for $\log A_t$ in Equation 71 then yields Equation AD:

$$\log P_t = \log \left(\frac{i_t}{1+i_t} \right) - \epsilon_t^D \log Y_t + \log M_t \tag{93}$$

Completing the proof. □

A.5 Proof of Theorem 3

Proof. We suppress dependence on t for ease of notation. We have χ_M and χ_A as a function of ω_1 from Lemma 1. We also know that:

$$\omega_1 = \frac{\sigma_{\mathcal{M}_i,z} + \sigma_{P,z}}{\sigma_z^2} \tag{94}$$

from Equation 51. As $z_i = \vartheta_i \left(\frac{i}{1+i}\right)^{\frac{1}{\gamma}} M^{\frac{1}{\gamma}} P^{\eta - \frac{1}{\gamma}}$ and $\mathcal{M}_i = \phi_i (z_i A)^{-1} \frac{i}{1+i} \frac{M}{P}$, we have that:

$$\begin{aligned}
\sigma_{\mathcal{M}_i,z} &= \mathrm{Cov}\left(-(1+\chi_A)\log A + (1-\chi_M)\log M, \left(\eta - \frac{1}{\gamma}\right)\chi_A \log A + \left(\frac{1}{\gamma} + \left(\eta - \frac{1}{\gamma}\right)\chi_M\right)\log M \right) \\
&= -\left(\eta - \frac{1}{\gamma}\right)\chi_A(1+\chi_A)\sigma_A^2 + (1-\chi_M)\left(\frac{1}{\gamma} + \left(\eta - \frac{1}{\gamma}\right)\chi_M\right)\sigma_M^2 \\
\sigma_{P,z} &= \mathrm{Cov}\left(\chi_A \log A + \chi_M \log M, \left(\eta - \frac{1}{\gamma}\right)\chi_A \log A + \left(\frac{1}{\gamma} + \left(\eta - \frac{1}{\gamma}\right)\chi_M\right)\log M \right) \\
&= \left(\eta - \frac{1}{\gamma}\right)\chi_A^2\sigma_A^2 + \chi_M\left(\frac{1}{\gamma} + \left(\eta - \frac{1}{\gamma}\right)\chi_M\right)\sigma_M^2 \\
\sigma_z^2 &= \sigma_\vartheta^2 + \left(\eta - \frac{1}{\gamma}\right)^2\chi_A^2\sigma_A^2 + \left(\frac{1}{\gamma} + \left(\eta - \frac{1}{\gamma}\right)\chi_M\right)^2\sigma_M^2
\end{aligned} \tag{95}$$

Thus:

$$\omega_1 = \frac{-(\eta - \frac{1}{\gamma})\chi_A \sigma_A^2 + (\frac{1}{\gamma} + (\eta - \frac{1}{\gamma})\chi_M)\sigma_M^2}{\sigma_\vartheta^2 + (\eta - \frac{1}{\gamma})^2 \chi_A^2 \sigma_A^2 + (\frac{1}{\gamma} + (\eta - \frac{1}{\gamma})\chi_M)^2 \sigma_M^2} \tag{96}$$

Note that the optimal ω_1 is common across all firms i. We may express this in fully reduced form as:

$$\omega_1 = T(\omega_1) = \frac{(\eta - \frac{1}{\gamma})\frac{\kappa_A}{1-\omega_1\left(\eta-\frac{1}{\gamma}\right)(1-\kappa_A)}\sigma_A^2 + (\frac{1}{\gamma} + (\eta - \frac{1}{\gamma})\frac{\kappa_M + \frac{\omega_1}{\gamma}(1-\kappa_M)}{1-\omega_1\left(\eta-\frac{1}{\gamma}\right)(1-\kappa_M)})\sigma_M^2}{\sigma_\vartheta^2 + (\eta - \frac{1}{\gamma})^2\left(\frac{\kappa_A}{1-\omega_1\left(\eta-\frac{1}{\gamma}\right)(1-\kappa_A)}\right)^2\sigma_A^2 + (\frac{1}{\gamma} + (\eta - \frac{1}{\gamma})\frac{\kappa_M + \frac{\omega_1}{\gamma}(1-\kappa_M)}{1-\omega_1\left(\eta-\frac{1}{\gamma}\right)(1-\kappa_M)})^2\sigma_M^2} \tag{97}$$

or

$$\omega_1 = T(\omega_1) = \frac{\frac{\left(\eta-\frac{1}{\gamma}\right)\kappa_A}{1-\omega_1\left(\eta-\frac{1}{\gamma}\right)(1-\kappa_A)}\sigma_A^2 + \frac{\frac{1}{\gamma}+\left(\eta-\frac{1}{\gamma}\right)\kappa_M}{1-\omega_1\left(\eta-\frac{1}{\gamma}\right)(1-\kappa_M)}\sigma_M^2}{\sigma_\vartheta^2 + \left(\frac{\left(\eta-\frac{1}{\gamma}\right)\kappa_A}{1-\omega_1\left(\eta-\frac{1}{\gamma}\right)(1-\kappa_A)}\right)^2\sigma_A^2 + \left(\frac{\frac{1}{\gamma}+\left(\eta-\frac{1}{\gamma}\right)\kappa_M}{1-\omega_1\left(\eta-\frac{1}{\gamma}\right)(1-\kappa_M)}\right)^2\sigma_M^2} \tag{98}$$

□

A.6 Proof of Proposition 2

Proof. We first establish equilibrium existence. First, we observe that T_t is a continuous function. The only possible points of discontinuity are: $\omega_{1,t}^M = \frac{1}{(\eta-\frac{1}{\gamma})(1-\kappa_t^M)}$ and $\omega_{1,t}^A = \frac{1}{(\eta-\frac{1}{\gamma})(1-\kappa_t^A)}$. However, at these points $\lim_{\omega_{1,t}\to\omega_{1,t}^M} T_t(\omega_{1,t}) = \lim_{\omega_{1,t}\to\omega_{1,t}^A} T_t(\omega_{1,t}) = T_t(\omega_{1,t}^M) = T_t(\omega_{1,t}^A) = 0$. Second, we observe that $\lim_{\omega_{1,t}\to-\infty} T_t(\omega_{1,t}) = \lim_{\omega_{1,t}\to\infty} T_t(\omega_{1,t}) = 0$. Consider now the function $W_t(\omega_{1,t}) = \omega_{1,t} - T_t(\omega_{1,t})$. This is a continuous function, $\lim_{\omega_{1,t}\to-\infty} W_t(\omega_{1,t}) = -\infty$, and $\lim_{\omega_{1,t}\to\infty} W_t(\omega_{1,t}) = \infty$. Thus, by the intermediate value theorem, there exists an $\omega_{1,t}^*$ such that $W_t(\omega_{1,t}^*) = 0$. By Theorem 3, $\omega_{1,t}^*$ defines a log-linear equilibrium.

We now show that there are at most five log-linear equilibria. For $\omega_{1,t} \neq \omega_{1,t}^A, \omega_{1,t}^M$ (neither of which can be a fixed point), we can rewrite Equation 29 as:

$$
\begin{aligned}
&\omega_{1,t}\Bigg[\sigma_{\vartheta,t}^2\left(1-\omega_{1,t}\left(\eta-\frac{1}{\gamma}\right)(1-\kappa_t^A)\right)^2\left(1-\omega_{1,t}\left(\eta-\frac{1}{\gamma}\right)(1-\kappa_t^M)\right)^2 \\
&\quad+\left(\sigma_{t|s}^A\right)^2\left(\eta-\frac{1}{\gamma}\right)\kappa_t^A\left(1-\omega_{1,t}\left(\eta-\frac{1}{\gamma}\right)(1-\kappa_t^M)\right)^2 \\
&\quad+\left(\sigma_{t|s}^M\right)^2\left(\frac{1}{\gamma}+\left(\eta-\frac{1}{\gamma}\right)\kappa_t^M\right)\left(1-\omega_{1,t}\left(\eta-\frac{1}{\gamma}\right)(1-\kappa_t^A)\right)^2\Bigg] \\
&=\left(\sigma_{t|s}^A\right)^2\left(\eta-\frac{1}{\gamma}\right)\kappa_t^A\left(1-\omega_{1,t}\left(\eta-\frac{1}{\gamma}\right)(1-\kappa_t^M)\right)^2\left(1-\omega_{1,t}\left(\eta-\frac{1}{\gamma}\right)(1-\kappa_t^A)\right) \\
&+\left(\sigma_{t|s}^M\right)^2\left(\frac{1}{\gamma}+\left(\eta-\frac{1}{\gamma}\right)\kappa_t^M\right)\left(1-\omega_{1,t}\left(\eta-\frac{1}{\gamma}\right)(1-\kappa_t^A)\right)^2\left(1-\omega_{1,t}\left(\eta-\frac{1}{\gamma}\right)(1-\kappa_t^M)\right)
\end{aligned}
\tag{99}
$$

This is a quintic polynomial in $\omega_{1,t}$, which has at most five real roots. Thus, by Theorem 3, there are at most five log-linear equilibria. □

A.7 Proof of Corollary 5

Proof. We drop time subscripts for ease of notation. Substituting $\eta = \frac{1}{\gamma}$ in Equation 29 yields:

$$
\omega_1 = \frac{\frac{1}{\gamma}}{\rho^2 + \left(\frac{1}{\gamma}\right)^2} \tag{100}
$$

Substituting this into Equation 24 yields:

$$
\epsilon_t^S = \gamma \frac{\kappa_t^M}{(1-\kappa_t^M)} + \frac{1}{\gamma\rho^2(1-\kappa_t^M)} \tag{101}
$$

□

A.8 Proof of Corollary 6

We drop time subscripts for ease of notation. The first statement follows directly from Equation 29. Furthermore, using Equation 29, as $\sigma_{t|s}^M \to \infty$, ω_1 must solve:

$$\begin{aligned}\omega_1 &= \frac{1-\omega_1\left(\eta-\frac{1}{\gamma}\right)\left(1-\kappa^M\right)}{\frac{1}{\gamma}+\left(\eta-\frac{1}{\gamma}\right)\kappa^M} = \frac{\gamma}{1+\left(\eta\gamma-1\right)\kappa^M} + \left(1-\frac{\eta\gamma}{1+(\eta\gamma-1)\kappa^M}\right)\omega_1 \\ &= \frac{1}{\eta}\end{aligned} \tag{102}$$

This proves the second statement. As $\sigma_{t|s}^A \to \infty$ and $\eta\gamma \neq 1$, ω_1 must solve:

$$\begin{aligned}\omega_1 &= \frac{1-\omega_1\left(\eta-\frac{1}{\gamma}\right)\left(1-\kappa^A\right)}{\left(\eta-\frac{1}{\gamma}\right)\kappa^A} = \frac{\gamma}{\left(\eta\gamma-1\right)\kappa^A} + \left(1-\frac{1}{\kappa^A}\right)\omega_1 \\ &= \frac{1}{\eta-\frac{1}{\gamma}}\end{aligned} \tag{103}$$

This proves the third statement.

A.9 Proof of Proposition 3

Proof. By Theorem 3, The map describing equilibrium $\omega_{1,t}$ is invariant to λ for $\lambda > 0$. Thus, $\mathcal{E}_t^S(\lambda)$ is constant for $\lambda > 0$. If $\lambda = 0$, there are potentially many equilibria in supply functions. Nevertheless, from the proof of Theorem 1, we have that firms set $p_{it}/P_t = \frac{\eta}{\eta-1}\mathcal{M}_{it} = \frac{\eta}{\eta-1}C_t^{\gamma}/A_t$ under any optimal supply function. This implies that $\frac{\eta}{\eta-1}C_t^{\gamma}/A_t = 1$, and so money has no real effects, which implies that $\epsilon_t^S = \infty$. □

B Supply Functions in Richer Economic Settings

In this appendix, we generalize the firm's partial-equilibrium supply schedule problem in four ways. First, we enrich both the firm's technology and input space by allowing for many inputs, decreasing returns to scale, and monopsony power. Second, we enrich the demand the firm faces by decoupling the own-price elasticity and the cross-price elasticity and allowing for non-isoelastic demand curves that feature endogenous markups (allowing for Marshall's Second and Third laws of demand). Third, we enrich the firm's decisionmaking by allowing the firm to choose additional non-price and non-quantity variables at a cost. This allows, for example, the firm to invest in improving the quality of its product. Finally, we enrich the firm's problem by introducing Calvo price stickiness. In all four cases, we characterize firms' optimal supply functions, show that our core insights generalize, and highlight the new economic features that each of these extensions introduces. In the interest of brevity, we leave embedding these generalizations in general equilibrium to future research, though it is clear to see how one could do this by embedding these characterizations in our general equilibrium model and leveraging the techniques from our main analysis.[20]

B.1 Multiple Inputs, Decreasing Returns, and Monopsony

In this section, we generalize our baseline model of supply function choice to allow for multiple inputs, decreasing returns, and monopsony. We find that: (i) supply functions remain endogenously log-linear and (ii) decreasing returns and monopsony flatten the optimal supply schedule.

Primitives. Consider the baseline model from Section 2 with two modifications. First, the production function uses multiple inputs with different input shares and possibly features decreasing returns-to-scale:

$$q = \Theta \prod_{i=1}^{I} x_i^{a_i} \tag{104}$$

where $x_i \in \mathbb{R}_+$, $a_i \geq 0$, and $\sum_{i=1}^{I} a_i \leq 1$. Moreover, suppose that the producer potentially has monopsony power and faces an upward-sloping factor price curve such that the price of acquiring any input i when the firm demands x_i units is given by $\tilde{p}_i(x_i) = p_{xi} x_i^{b_i - 1}$, where $p_{xi} \in \mathbb{R}_{++}$ and $b_i \geq 1$. The case of no monopsony, or price-taking in the input market, occurs

[20] The only complication with endogenous markups would be the endogenous non-log-linearity of the optimal supply curve. This would have to be dealt with via either approximation arguments similar to those we adopt in our extension to allow for price stickiness or numerical methods, or both.

when $b_i = 1$. Thus, the cost of acquiring each type of input is given by:

$$c_i(x_i) = p_{xi} x_i^{b_i} \tag{105}$$

The firm believes that $(\Psi, P, \Lambda, \Theta, p_x)$ is jointly log-normal.

The Firm's Problem. We begin by solving the firm's cost minimization problem:

$$K(q; \Theta, p_x) = \min_x \sum_{i=1}^{I} p_{x_i} x_i^{b_i} \quad \text{s.t.} \quad q = \Theta \prod_{i=1}^{I} x_i^{a_i} \tag{106}$$

This has first-order condition given by:

$$\lambda = \frac{b_i p_{xi}}{a_i} x_i^{b_i} q^{-1} \tag{107}$$

Which implies that:

$$K(q; \Theta, p_x) = \lambda q \sum_{i=1}^{I} \frac{a_i}{b_i} \tag{108}$$

Moreover, fixing i, the FOC implies that we may write for all $j \neq i$:

$$x_j = \left(\frac{\frac{b_i p_{x_i}}{a_i}}{\frac{b_j p_{x_j}}{\alpha_j}} \right)^{\frac{1}{b_j}} x_i^{\frac{b_i}{b_j}} \tag{109}$$

By substituting this into the production function we have that:

$$q = \Theta x_i^{a_i + b_i \sum_{j \neq i} \frac{a_j}{b_j}} \prod_{j \neq i} \left(\frac{\frac{b_i p_{x_i}}{a_i}}{\frac{b_j p_{x_j}}{\alpha_j}} \right)^{\frac{\alpha_j}{b_j}} \tag{110}$$

which implies that:

$$x_i = \left(\frac{q}{\Theta \prod_{j \neq i} \left(\frac{\frac{b_i p_{x_i}}{a_i}}{\frac{b_j p_{x_j}}{\alpha_j}} \right)^{\frac{\alpha_j}{b_j}}} \right)^{\frac{1}{a_i + b_i \sum_{j \neq i} \frac{a_j}{b_j}}} \tag{111}$$

Returning to the FOC, we have that the Lagrange multiplier is given by:

$$\lambda = q^{-1+\frac{1}{\sum_{i=1}^{I} \frac{a_i}{b_i}}} \frac{b_i p_{xi}}{a_i} \left(\Theta \prod_{j \neq i} \left(\frac{\frac{b_i p_{x_i}}{a_i}}{\frac{b_j p_{x_j}}{\alpha_j}} \right)^{\frac{\alpha_j}{b_j}} \right)^{\frac{-1}{\sum_{i=1}^{I} \frac{a_j}{b_j}}} \tag{112}$$

Which then yields the cost function:

$$K(q; \Theta, p_x) = \mathcal{M} P q^{\frac{1}{\delta}} \tag{113}$$

where:

$$\delta = \sum_{i=1}^{I} \frac{a_i}{b_i} \quad \text{and} \quad \mathcal{M} = P^{-1} \left(\Theta \prod_{i=1}^{I} \left(\frac{b_i p_{x_i}}{\alpha_i} \right)^{\frac{\alpha_j}{b_j}} \right)^{\frac{1}{\sum_{i=1}^{I} \frac{a_i}{b_i}}} \sum_{i=1}^{I} \frac{a_i}{b_i} \tag{114}$$

and we observe that $\mathcal{M}$ is log-normal given the joint log-normality of (Θ, p_x).

Turning to the firm's payoff function, we therefore have:

$$\mathbb{E} \left[\Lambda \left(\frac{p}{P} q - \mathcal{M} q^{\frac{1}{\delta}} \right) \right] \tag{115}$$

Thus, the problem with multiple inputs, monopsony, and decreasing returns modifies the firms' original payoff by only introducing the parameter δ. Helpfully, observe that $\delta = 1$ when: (i) there are constant returns to scale $\sum_{i=1}^{I} a_i = 1$ and (ii) there is no monopsony $b_i = 1$ for all i.

Given this, we can write the firm's objective as:

$$J(\hat{p}) = \int_{\mathbb{R}^4_{++}} \Lambda \left(\frac{\hat{p}(z)^{1-\eta}}{P} z - \mathcal{M} z^{\frac{1}{\delta}} \hat{p}(z)^{-\frac{\eta}{\delta}} \right) \mathrm{d}G\left(\Lambda, P, \mathcal{M}, z\right) \tag{116}$$

And, as before, we study the problem:

$$\sup_{\hat{p}: \mathbb{R}_+ \to \mathbb{R}_{++}} J(\hat{p}) \tag{117}$$

By doing this, we obtain a modified formula for the optimal supply function:

Proposition 4 (Optimal Supply Schedule With Multiple Inputs, Decreasing Returns, and Monopsony)**.** *Any optimal supply schedule is almost everywhere given by:*

$$f(p, q) = \log p - \frac{\omega_0 - \log \delta}{1 - \eta \omega_1} - \frac{\omega_1 + \frac{1-\delta}{\delta}}{1 - \eta \omega_1} \log q \tag{118}$$

where ω_0 and ω_1 are the same as those derived in Theorem 1. Thus, the optimal inverse *supply elasticity is given by:*

$$\hat{\alpha}_1 = \frac{\eta\sigma_P^2 + \sigma_{\mathcal{M},\Psi} + \sigma_{P,\Psi} + \eta\sigma_{\mathcal{M},P}}{\sigma_\Psi^2 - \eta\sigma_{\mathcal{M},\Psi} + \eta\sigma_{P,\Psi} - \eta^2\sigma_{\mathcal{M},P}} \left(1 + \frac{1-\delta}{\delta} \frac{\sigma_\Psi^2 + \eta^2\sigma_P^2 + 2\eta\sigma_{\Psi,P}}{\sigma_{\mathcal{M},\Psi} + \eta\sigma_{\mathcal{M},P} + \sigma_{P,\Psi} + \eta\sigma_P^2}\right) \tag{119}$$

Proof. Applying the same variational arguments as in the Proof of Theorem 1, we obtain that $\hat{p}(t)$ must solve:

$$(\eta - 1)\mathbb{E}[\Lambda P^{-1}|z = t]t\hat{p}(t)^{-\eta} = \frac{\eta}{\delta}\mathbb{E}[\Lambda\mathcal{M}|z = t]t^{\frac{1}{\delta}}\hat{p}(z)^{-\frac{\eta}{\delta}-1} \tag{120}$$

Which yields:

$$\hat{p}(t) = \left(\delta^{-1}\frac{\eta}{\eta-1}\frac{\mathbb{E}[\Lambda\mathcal{M}|z = t]}{\mathbb{E}[\Lambda P^{-1}|z = t]}\right)^{\frac{1}{1+\eta\left(\frac{1-\delta}{\delta}\right)}} t^{\frac{\frac{1-\delta}{\delta}}{1+\eta\left(\frac{1-\delta}{\delta}\right)}} \tag{121}$$

Thus, we have that:

$$\log p = \frac{1}{1+\eta\left(\frac{1-\delta}{\delta}\right)}(\omega_0 - \log\delta) + \frac{1}{1+\eta\left(\frac{1-\delta}{\delta}\right)}\left(\omega_1 + \frac{1-\delta}{\delta}\right)\log z \tag{122}$$

where ω_0 and ω_1 are as in Theorem 1. Rewriting as a supply schedule, we obtain:

$$\log p = \frac{\frac{1}{1+\eta\left(\frac{1-\delta}{\delta}\right)}(\omega_0 - \log\delta)}{1 - \frac{\eta}{1+\eta\left(\frac{1-\delta}{\delta}\right)}\left(\omega_1 + \frac{1-\delta}{\delta}\right)} + \frac{\frac{1}{1+\eta\left(\frac{1-\delta}{\delta}\right)}\left(\omega_1 + \frac{1-\delta}{\delta}\right)}{1 - \frac{\eta}{1+\eta\left(\frac{1-\delta}{\delta}\right)}\left(\omega_1 + \frac{1-\delta}{\delta}\right)}\log q \tag{123}$$

Which reduces to the claimed formula. □

Thus, when the supply curve is initially upward-sloping ($\omega_1 \in [0, \eta^{-1}]$), the introduction of decreasing returns and/or monopsony unambiguously increases the supply elasticity and makes firms closer to quantity-setting.

B.2 Beyond Isoelastic Demand

Isoelastic demand imposes both that the firm's own price elasticity of demand and its cross-price elasticity of demand are constant. In this appendix, we show how to derive optimal supply functions in closed form when the firm's own price elasticity of demand varies. This allows the demand curve to satisfy Marshall's second law of demand that the price elasticity of demand is increasing in the price as well as Marshall's third law of demand that the rate of increase of the price elasticity goes down with the price. We show that uncertainty about demand, prices, and marginal costs continue to operate in a very similar fashion. However, due to endogeneity of the optimal markup, the optimal supply schedule now ceases to be log-linear.

To capture these features, suppose that demand is *multiplicatively separable*: $d(p, \Psi, P) = z(\Psi, P)\phi(p)$ for some function ϕ such that $p\phi''(p)/\phi'(p) < -2$. This latter condition is satisfied by isoelastic demand exactly under the familiar condition that $\eta > 1$ and ensures the existence of a unique optimal price. We further assume that $z(\Psi, P) = \nu_0 \Psi^{\nu_1} P^{\nu_2}$ for $\nu_0, \nu_1, \nu_2 \in \mathbb{R} \setminus \{0\}$. This makes firms' uncertainty about the location of their demand curve log-normal. This assumption does rule out non-separable demand, such as the demand system proposed by Kimball (1995). However, it is important to note that this demand system is motivated by evidence on the firm's own price elasticity, which is governed by ϕ, and not the cross-price elasticity, which is governed by ν_2. Thus, our proposed demand system is equally able to capture facts about the firms' own price elasticity as the one proposed in Kimball (1995), or the richer structures proposed by Fujiwara and Matsuyama (2022) and Wang and Werning (2022).

Under this demand system, we can derive a modified formula for the optimal supply curve which is now no longer log-linear, but continues to be governed by similar forces:

Proposition 5. *If demand is multiplicatively separable, then any optimal supply function is almost everywhere given by:*

$$f(p, q) = \log q + \hat{\alpha}_0 - \log \left(\phi(p) \left\{ p \left[1 + \frac{\phi(p)}{p\phi'(p)} \right] \right\}^{\frac{1}{\hat{\omega}_1}} \right) \tag{124}$$

where:

$$\hat{\omega}_1 = \frac{\nu_1(\sigma_{\mathcal{M},\Psi} + \sigma_{P,\Psi}) + \nu_2(\sigma_P^2 + \sigma_{\mathcal{M},P})}{\nu_1^2 \sigma_\Psi^2 + \nu_2^2 \sigma_P^2 + 2\nu_1\nu_2\sigma_{\Psi,P}} \tag{125}$$

Proof. Applying the same variational arguments as in Theorem 1, we obtain that:

$$\hat{p}(z) + \frac{\phi(\hat{p}(z))}{\phi'(\hat{p}(z))} = \frac{\mathbb{E}[\Lambda \mathcal{M} | z]}{\mathbb{E}[\Lambda P^{-1} | z]} \tag{126}$$

where the condition $p\phi''(p)/\phi'(p) < -2$ yields strict concavity of the objective and makes $\hat{p}(z)$ the unique maximizer. Taking logarithms of both sides and evaluating the conditional expectations as per Theorem 1, we obtain that:

$$\log \left(\hat{p}(z) \left[1 + \frac{\phi(\hat{p}(z))}{\hat{p}(z)\phi'(\hat{p}(z))} \right] \right) = \hat{\omega}_0 + \hat{\omega}_1 \log z \tag{127}$$

where $\hat{\omega}_1 = \frac{\sigma_{\mathcal{M},z} + \sigma_{P,z}}{\sigma_z^2}$, which yields Equation 125. Using $\log z = \log q - \log \phi(p)$ and rearranging yields Equation 124. □

Demand uncertainty and price uncertainty enter the same way as before, via $\hat{\omega}_1$, and the intuition is the same. However, there are now two distinct notions of market power and they therefore operate in a more subtle way. First, consider the role of the cross-price elasticity of demand ν_2. When ν_2 is higher, the firm's price is *ex post* more responsive to changes in others' prices. Second, consider the role of the own-price elasticity of demand $\left(\frac{p\phi'(p)}{\phi(p)}\right)^{-1}$. This induces non-linearity of the optimal supply schedule to the extent that it is not constant. This is because the firm's optimal markup changes as it moves along its demand curve.

B.3 Additional Choice Variables

Our approach of studying firms' supply functions has thus far focused on firms that choose prices and quantities. However, it is natural to imagine that firms can make richer choices, such as deciding what quality or type of product they will sell. In this appendix, we generalize our characterization of firms' optimal supply functions to incorporate additional choice margins. We find that supply functions remain log-linear conditional on these other choices. We also show how to characterize the optimal values of these other choices given this fact.

To model additional choice margins, suppose that the firm, in addition to its price and quantity decisions, chooses a vector of non-quantity decisions $x \in X \subseteq \mathbb{R}^n$. These decisions are made at the beginning of the period and potentially affect the joint distribution of $(\Lambda, P, \mathcal{M}, \Psi)$ via the map $G : X \to \Delta(\mathbb{R}^4_+)$. We suppose that choices of $x \in X$ lead to a dollar cost to the firm of $C(x)$. To see how this framework accommodates quantity investments, suppose that $X \subseteq \mathbb{R}$ and $x \in X$ represents the quality of the good. Investing in different qualities comes at a cost. Moreover, higher quality might increase both the mean of firms' demand Ψ and the mean of firms' marginal costs $\mathcal{M}$.

We now characterize firms' optimal supply function decisions in this framework. We let $H(f, x)$ denote the joint distribution over $(\Lambda, P, \mathcal{M}, \Psi, p, q)$ induced by a supply function $f : \mathbb{R}^2_{++} \to \mathbb{R}$ and other decisions x. With this, the firm's problem of optimal supply function and other decisions is given by:

$$\sup_{x \in X, f:\mathbb{R}^2_{++} \to \mathbb{R}} \mathbb{E}_{H(f,x)} \left[\Lambda \left(\frac{p}{P} - \mathcal{M}\right) q\right] - \mathbb{E}_{H(f,x)}[\Lambda] C(x) \tag{128}$$

This can be split into two optimization problems. First, for every choice of $x \in X$, we solve for the optimal supply function f_x:

$$V(x) = \sup_{f:\mathbb{R}^2_{++} \to \mathbb{R}} \mathbb{E}_{H(f,x)} \left[\Lambda \left(\frac{p}{P} - \mathcal{M}\right) q\right] \tag{129}$$

Second, we can compute the optimal choice of $x \in X$ by solving:

$$\sup_{x \in X} V(x) - \mathbb{E}_{H(f,x)}[\Lambda]C(x) \tag{130}$$

By identical arguments to those of Theorem 1 (simply index G by x up to Equation 44), we immediately obtain that under the optimal prices in demand state $z = t$ must be given by:

$$p_x(t) = \frac{\eta}{\eta - 1} \frac{\mathbb{E}_{G(x)}[\Lambda \mathcal{M} | z = t]}{\mathbb{E}_{G(x)}[\Lambda P^{-1} | z = t]} \tag{131}$$

If we further assume that $G(x)$ is a multivariate log-normal distribution with mean μ_x and variance-covariance matrix Σ_x, then we obtain (by identical arguments to those in Theorem 1) that the optimal supply function for a fixed choice of $x \in X$ obeys the following Proposition, the proof of which follows immediately from that of Theorem 1.

Proposition 6 (Supply Function Choice When Firms Choose More Than Prices and Quantities)**.** *If for $x \in X$ the distribution $G(x)$ is multivariate normal, then the optimal supply function is given by:*

$$f_x(p, q) = \log p - \alpha_{0,x} - \alpha_{1,x} \log q \tag{132}$$

where $\alpha_{0,x}$ and $\alpha_{1,x}$ follow exactly the formulae derived in Theorem 1, where all appropriate means and variances are computed under the distribution $G(x)$.

From this, we observe that Theorem 1 carries as written in this extended setting. In particular, supply functions remain log-linear and the same variances and covariances govern their elasticity. The new feature here is that the choice of x can affect both the intercept and the slope of the optimal supply function. In this way, the choice of x can have a non-trivial effect on firms' optimal pricing and production decisions.

With this, we can now explicitly characterize the value of any choice of x and thereby solve for the optimal choice of x. Concretely, we have that:

$$V(x) = \int_{\mathbb{R}^4_{++}} \Lambda \frac{z}{P} p_x(z)^{1-\eta} \mathrm{d}G_x(\Lambda, P, \mathcal{M}, \Psi) - \int_{\mathbb{R}^4_{++}} \Lambda z \mathcal{M} p_x(z)^{-\eta} \mathrm{d}G_x(\Lambda, P, \mathcal{M}, \Psi) \tag{133}$$

Substituting Equation 131, this becomes:

$$\begin{aligned} V(x) = &\int_{\mathbb{R}^4_{++}} \Lambda \frac{z}{P} \exp\{(1-\eta)\omega_{0,x}\} z^{(1-\eta)\omega_{1,x}} \mathrm{d}G_x(\Lambda, P, \mathcal{M}, \Psi) \\ &- \int_{\mathbb{R}^4_{++}} \Lambda z \mathcal{M} \exp\{-\eta \omega_{0,x}\} z^{-\eta \omega_{1,x}} \mathrm{d}G_x(\Lambda, P, \mathcal{M}, \Psi) \end{aligned} \tag{134}$$

where $\omega_{0,x}$ and $\omega_{1,x}$ have the same formulae as those in the proof of Theorem 1 (with all means, variances, and covariances indexed by x). Exploiting joint log-normality of G_x, we can evaluate these integrals to obtain:

$$\begin{aligned} V(x) = \exp &\left\{ (1-\eta)\omega_{0,x} + \mu_{\Lambda,x} - \mu_{P,x} + (1+\omega_{1,x}(1-\eta))\mu_{z,x} + \frac{1}{2}\sigma^2_{R,x} \right\} \\ &- \exp\left\{ -\eta\omega_{0,x} + \mu_{\Lambda,x} + \mu_{\mathcal{M},x} + (1-\eta\omega_{1,x})\mu_{z,x} + \frac{1}{2}\sigma^2_{C,x} \right\} \end{aligned} \tag{135}$$

where:

$$\begin{aligned} \sigma^2_{R,x} &= \mathbb{V}_x \left[\log \Lambda - \log P + (1+\omega_{1,x}(1-\eta)) \log z \right] \\ \sigma^2_{C,x} &= \mathbb{V}_x \left[\log \Lambda + \log \mathcal{M} + (1-\eta\omega_{1,x}) \log z \right] \end{aligned} \tag{136}$$

With this, solving for the optimal choice of $x \in X$ reduces to solving Equation 130 using this V and given the exogenous function C.

We conclude by characterizing the optimal x in a simple example.

Example 1. *Suppose that quality can be improved at some ex ante cost and that quality affects how much consumers demand the product and nothing else. Formally, suppose that* $C(x) = \frac{\zeta}{2}x^2$, $\mu_{\Psi,x} = \mu_\Psi + \log x$, $\sigma_x \equiv \sigma$ *and* μ_x *is invariant to* x *except for* $\mu_{\Psi,x}$. *In the previous formulae, observe that* $(\omega_{1,x}, \sigma^2_{C,x}.\sigma^2_{R,x}, \mu_{\Lambda,x}, \mu_{P,x}, \mu_{\mathcal{M},x})$ *are invariant to* x. *Thus, observing that* $\omega_{0,x}$ *is affine in* $\log x$, *we obtain that* V *is linear in* x, *i.e.,* $V(x) = Kx$ *for some* $K > 0$. *It follows that the optimal choice is given by* $x^* = \frac{K}{\zeta}$.

This example shows that the approach followed in this appendix can be practically useful in extending the supply function approach to consider firms that can choose additional variables.

B.4 Supply Functions with Sticky Prices

In our main analysis, we allowed firms to change their prices every period to emphasize the new economic features that supply functions generate. At the same time, our approach can be augmented to include price stickiness. In this appendix, we show how to solve for the optimal supply function when firms are subject to Calvo pricing.

Firms are as in our main analysis, except their prices are sticky each period with probability $\theta \in [0, 1]$. For this appendix, we apply the standard second-order approximation to firms' profits and write the flow profit of the firm as:

$$-B(\log p - \log p^{**})^2 \tag{137}$$

where we recall that $p^{**} = \frac{\eta}{\eta-1}\mathcal{M}P$ and $B > 0$ is the curvature of the profit function. Under this approximation, the firm's lifetime loss from setting price p_t at date t is given by:

$$\mathcal{L}(p_t) = B\sum_{j=0}^{\infty}(\beta\theta)^j(\log p_t - \log p^{**}_{t+j})^2 \tag{138}$$

As in our main analysis, at date t, a price-resetting firm chooses a supply function f_t and they will produce at the price and quantity such that the $f_t(p_t, q_t) = 0$ locus intersects the demand curve $\log z_t = \log q_t - \eta \log p_t$. By applying similar arguments to those of Theorem 1, we obtain the following characterization of the optimal supply function:

Proposition 7 (Optimal Supply Function with Price Stickiness)**.** *For a firm with Calvo stickiness parameter $\theta \in [0,1]$ and discount factor $\beta \in [0,1)$, any optimal supply curve is almost everywhere given by:*

$$f_t(p_t, q_t) = \log p_t - \alpha_{0,t} - \alpha_{1,t}\log q_t \tag{139}$$

where the slope of the optimal price-quantity locus, $\alpha_{1,t} \in \overline{\mathbb{R}}$, is given by:

$$\alpha_{1,t} = \frac{\hat{\omega}_{1,t}}{1 - \eta\hat{\omega}_{1,t}} \tag{140}$$

where:

$$\hat{\omega}_{1,t} = (1-\beta\theta)\sum_{j=0}^{\infty}(\beta\theta)^j\omega_{1,t,j} \tag{141}$$

and:

$$\omega_{1,t,j} = \frac{\sigma_{\mathcal{M}_{t+j},z_t} + \sigma_{P_{t+j},z_t}}{\sigma^2_{z_t}} \tag{142}$$

Proof. We first characterize the optimal z_t-measurable price, $\hat{p}_t(z_t)$. Taking the first-order condition of the firm's expected loss, we have that:

$$\log \hat{p}_t(z_t) = (1-\beta\theta)\mathbb{E}_t\left[\sum_{j=0}^{\infty}(\beta\theta)^j \log p^{**}_{t+j} \mid z_t\right] \tag{143}$$

We moreover have that:

$$\mathbb{E}_t[\log p^{**}_{t+j}|z_t] = \mathbb{E}_t\left[\log\frac{\eta}{\eta-1} + \log P_{t+j} + \log\mathcal{M}_{t+j} \mid z_t\right] = \omega_{0,t,j} + \omega_{1,t,j}z_t \tag{144}$$

where:

$$\begin{aligned}\omega_{1,t,j} &= \frac{\sigma_{\mathcal{M}_{t+j},z_t} + \sigma_{P_{t+j},z_t}}{\sigma^2_{z_t}} \\ \omega_{0,t,j} &= \log\frac{\eta}{\eta-1} + \mu_{\mathcal{M}_{t+j}} + \mu_{P_{t+j}} - \omega_{1,t,j}\mu_{z_t}\end{aligned} \tag{145}$$

which are both deterministic functions of t and j. Substituting this into the formula for the firm's optimal z_t-measureable price, we obtain that:

$$\begin{aligned}\log\hat{p}_t(z_t) &= (1-\beta\theta)\sum_{j=0}^{\infty}(\beta\theta)^j\omega_{0,t,j} + \left[(1-\beta\theta)\sum_{j=0}^{\infty}(\beta\theta)^j\omega_{1,t,j}\right]z_t \\ &= \hat{\omega}_{0,t} + \hat{\omega}_{1,t}z_t\end{aligned} \tag{146}$$

Using the fact that the firm's demand curve is $\log z_t = \log q_t - \eta\log p_t$, we obtain that $\log p_t = \alpha_{0,t} + \alpha_{1,t}\log q_t$ with $\alpha_{0,t} = \frac{\hat{\omega}_{0,t}}{1-\eta\omega_{\hat{1},t}}$ and $\alpha_{1,t} = \frac{\hat{\omega}_{1,t}}{1-\eta\hat{\omega}_{1,t}}$, completing the proof. □

From this, we observe that price stickiness modifies the slope of the firm's optimal supply function, but it remains optimally log-linear (at least under the quadratic approximation to the firm's flow profit that is standard in dynamic Calvo pricing models). The firm's optimal supply elasticity now incorporates how much the firm learns from its demand today about the whole sequence of its current and future nominal marginal costs. The inference that it performs about its date $t+j$ marginal costs from today's demand is captured by $\omega_{1,t,j}$, which is precisely the OLS regression coefficient that one obtains from regressing nominal marginal costs at date $t+j$ on demand at date t. In deciding its optimal price today, the firm then must weigh its inference about future nominal marginal costs by how much it cares about the future j periods from now (β^j) and how likely its price today is to prevail in j periods (θ^j). This weighting yields $\hat{\omega}_{1,t}$, which captures the overall responsiveness of the price today to demand today. Once this has been obtained, we can convert this into the slope of the optimal supply curve as we did in our main analysis via the transformation $\hat{\omega}_{1,t} \mapsto \frac{\hat{\omega}_{1,t}}{1-\eta\hat{\omega}_{1,t}} \equiv \alpha_{1,t}$.

This analysis highlights that the supply function approach is not a replacement for sticky price models, but rather represents a different approach to modelling how firms that can reset their prices do so. While we abstract from sticky prices in our main analysis to make plain the new modelling implications of supply functions, the analysis of this appendix demonstrates that it is practically simple to combine our supply function approach with canonical approaches to modelling sticky prices.

C Allowing for Correlated Aggregate Shocks

In this extension, we allow for the shocks to the money supply and aggregate productivity to be correlated. Specifically, we assume that, conditional on outcomes in period $t-1$ that $(\log A_t, \log M_t)$ is jointly normally distributed. Our main analysis assumes that $\log A_t$ and $\log M_t$ are uncorrelated. Allowing for correlation modifies firms' conditional expectations of the aggregate shocks to the following:

$$\begin{aligned} \mathbb{E}_{i,t}[\log A_t] &= \text{cons}_t + \kappa_t^A s_{it}^A + \tilde{\kappa}_t^A s_{it}^M \\ \mathbb{E}_{i,t}[\log M_t] &= \text{cons}_t + \kappa_t^M s_{it}^M + \tilde{\kappa}_t^M s_{it}^A \end{aligned} \tag{147}$$

where cons_t are terms independent of the realized shocks at date t, and $(\kappa_t^A, \tilde{\kappa}_t^A, \kappa_t^M, \tilde{\kappa}_t^M)$ are the Kalman gains.

In this extended setting, Theorem 1 on firms' optimal supply functions holds as written. Theorem 2 on the AS/AD representation holds with modified formulae for the slopes of the aggregate demand and aggregate supply curves as the guess and verify argument must be modified to account for the new formulae for firms' expectations of aggregate shocks. Performing this modification, we obtain the following:

Proposition 8. *There exists a unique log-linear temporary equilibrium that is described by an "Aggregate Demand/Aggregate Supply" model in which the slope of the aggregate supply curve is given by:*

$$\epsilon_t^S = \gamma \frac{\chi_{M,t}}{1 - \chi_{M,t}} \tag{148}$$

where:

$$\chi_{M,t} = \frac{\kappa_t^M + (1-\kappa_t^M)\frac{1}{\gamma}\omega_{1,t} + \tilde{\kappa}_t^A \left(-1 - \left(\eta - \frac{1}{\gamma}\omega_{1,t} \frac{-\kappa_t^A + \left(1 - \frac{1}{\gamma}\omega_{1,t}\right)\tilde{\kappa}_t^M}{1-\omega_{1,t}\left(\eta - \frac{1}{\gamma}\right)(1-\kappa_t^A)}\right)\right)}{1 - \omega_{1,t}\left(\eta - \frac{1}{\gamma}\right)\left((1-\kappa_t^M) - \frac{\omega_{1,t}\left(\eta - \frac{1}{\gamma}\right)\tilde{\kappa}_t^M}{1-\omega_{1,t}\left(\eta-\frac{1}{\gamma}\right)(1-\kappa_t^A)}\tilde{\kappa}_t^A\right)} \tag{149}$$

Proof. As in the proof of Theorem 2, we will guess and verify that (dropping t subscripts for compactness):

$$\log P = \chi_0 + \chi_A \log A + \chi_M \log M \tag{150}$$

The same arguments as Theorem 2 imply that we must compute:

$$p_i = \frac{\eta}{\eta - 1} \frac{\mathbb{E}_i\left[\phi(z_i A)^{-1}\left(\vartheta_i I^{-\frac{1}{\gamma}} P^{-\frac{1}{\gamma}} M^{\frac{1}{\gamma}} P^{\eta}\right)^{1-\eta\omega_1}\right]}{\mathbb{E}_i\left[I^{1-\frac{1}{\gamma}(1+\omega_1-\eta\omega_1)} M^{\frac{1}{\gamma}(1+\omega_1-\eta\omega_1)-1} \vartheta^{1+\omega_1-\eta\omega_1} P^{\left(\eta-\frac{1}{\gamma}\right)(1+\omega_1-\eta\omega_1)}\right]} \times \left(\vartheta_i I^{-\frac{1}{\gamma}} M^{\frac{1}{\gamma}} P^{\eta-\frac{1}{\gamma}}\right)^{\omega_1} \tag{151}$$

Moreover, the same arguments as Theorem 2 imply that:

$$\log P = \mathbb{E}[\log p_i] + \frac{1}{2(1-\eta)} \mathrm{Var}((1-\eta)\log p_i) + \mathrm{cons} \tag{152}$$

We now compute the numerator, denominator and multiplicative terms in the firm's pricing equation that obtain under their chosen supply function:

$$\begin{aligned} &\log \mathbb{E}_i\left[\phi(z_i A)^{-1}\left(\vartheta_i I^{-\frac{1}{\gamma}} P^{-\frac{1}{\gamma}} M^{\frac{1}{\gamma}} P^{\eta}\right)^{1-\eta\omega_1}\right] = \mathrm{cons} \\ &+ \left(-1 + \left(\eta - \frac{1}{\gamma}\right)(1-\eta\omega_1)\chi_A\right)\mathbb{E}_i[\log A] \\ &+ \left(\frac{1}{\gamma}(1-\eta\omega_1) + \left(\eta - \frac{1}{\gamma}\right)(1-\eta\omega_1)\chi_M\right)\mathbb{E}_i[\log M] \end{aligned} \tag{153}$$

$$\begin{aligned} &\log \mathbb{E}_i\left[I^{1-\frac{1}{\gamma}(1+\omega_1-\eta\omega_1)} M^{\frac{1}{\gamma}(1+\omega_1-\eta\omega_1)-1} \vartheta^{1+\omega_1-\eta\omega_1} P^{\left(\eta-\frac{1}{\gamma}\right)(1+\omega_1-\eta\omega_1)}\right] = \mathrm{cons} \\ &+ \left(\eta - \frac{1}{\gamma}\right)(1-\eta\omega_1+\omega_1)\chi_A \mathbb{E}_i[\log A] \\ &+ \left(-1 + \frac{1}{\gamma}(1-\eta\omega_1+\omega_1) + \left(\eta - \frac{1}{\gamma}\right)(1-\eta\omega_1+\omega_1)\chi_M\right)\mathbb{E}_i[\log M] \end{aligned} \tag{154}$$

$$\begin{aligned} &\log\left(\vartheta_i I^{-\frac{1}{\gamma}} M^{\frac{1}{\gamma}} P^{\eta-\frac{1}{\gamma}}\right)^{\omega_1} = \mathrm{cons} \\ &+ \omega_1\left(\eta - \frac{1}{\gamma}\right)\chi_A \log A + \omega_1\left(\frac{1}{\gamma} + \left(\eta - \frac{1}{\gamma}\right)\chi_M\right)\log M \end{aligned} \tag{155}$$

From this, we have that:

$$\begin{aligned} &\log p_i = \mathrm{cons} \\ &+ \left(-1 - \left(\eta - \frac{1}{\gamma}\right)\omega_1\chi_A\right)\mathbb{E}_i[\log A] + \omega_1\left(\eta - \frac{1}{\gamma}\right)\chi_A \log A \\ &+ \left(1 - \frac{1}{\gamma}\omega_1 - \left(\eta - \frac{1}{\gamma}\right)\omega_1\chi_M\right)\mathbb{E}_i[\log M] + \omega_1\left(\frac{1}{\gamma} + \left(\eta - \frac{1}{\gamma}\right)\chi_M\right)\log M \end{aligned} \tag{156}$$

Aggregating this according to the aggregation formula, we obtain:

$$
\begin{aligned}
\log P &= \text{cons} \\
&+ \left(-1 - \left(\eta - \frac{1}{\gamma}\right)\omega_1 \chi_A\right) \overline{\mathbb{E}}[\log A] + \omega_1 \left(\eta - \frac{1}{\gamma}\right) \chi_A \log A \\
&+ \left(1 - \frac{1}{\gamma}\omega_1 - \left(\eta - \frac{1}{\gamma}\right)\omega_1 \chi_M\right) \overline{\mathbb{E}}[\log M] + \omega_1 \left(\frac{1}{\gamma} + \left(\eta - \frac{1}{\gamma}\right)\chi_M\right) \log M
\end{aligned}
\tag{157}
$$

Up to this point, everything is the same as Theorem 2. The presence of correlated aggregate shocks now changes the formulae for $(\overline{\mathbb{E}}[\log A], \overline{\mathbb{E}}[\log M])$. These are now given by:

$$
\begin{aligned}
\overline{\mathbb{E}}[\log A] &= \text{cons} + \kappa^A \log A + \tilde{\kappa}^A \log M \\
\overline{\mathbb{E}}[\log A] &= \text{cons} + \kappa^M \log M + \tilde{\kappa}^M \log A
\end{aligned}
\tag{158}
$$

Plugging these into the formula for the aggregate price level and collecting terms:

$$
\begin{aligned}
\log P &= \text{cons} \\
&+ \left(\omega_1 \left(\eta - \frac{1}{\gamma}\right)\chi_A + \left(-1 - \left(\eta - \frac{1}{\gamma}\right)\omega_1\chi_A\right)\kappa^A + \left(1 - \frac{1}{\gamma}\omega_1 - \left(\eta - \frac{1}{\gamma}\right)\omega_1\chi_M\right)\tilde{\kappa}^M\right)\log A \\
&+ \left(\omega_1 \left(\frac{1}{\gamma} + \left(\eta - \frac{1}{\gamma}\right)\chi_M\right) + \left(1 - \frac{1}{\gamma}\omega_1 - \left(\eta - \frac{1}{\gamma}\right)\omega_1\chi_M\right)\kappa^M + \left(-1 - \left(\eta - \frac{1}{\gamma}\right)\omega_1\chi_A\right)\tilde{\kappa}^A\right)\log M
\end{aligned}
\tag{159}
$$

Thus, by matching coefficients and simplifying, we have that:

$$
\begin{aligned}
\chi_A &= -\kappa^A + (1 - \kappa^A)\left(\eta - \frac{1}{\gamma}\right)\omega_1\chi_A + \left(1 - \frac{1}{\gamma}\omega_1 - \left(\eta - \frac{1}{\gamma}\right)\omega_1\chi_M\right)\tilde{\kappa}^M \\
\chi_M &= \kappa^M + (1 - \kappa^M)\frac{1}{\gamma}\omega_1 + (1 - \kappa^M)\left(\eta - \frac{1}{\gamma}\right)\omega_1\chi_M + \left(-1 - \left(\eta - \frac{1}{\gamma}\right)\omega_1\chi_A\right)\tilde{\kappa}^A
\end{aligned}
\tag{160}
$$

We can now solve this linear system of equations in (χ^A, χ^M). To do this, we first solve for χ_A as a function of χ_M:

$$
\chi_A = \frac{-\kappa^A + \left(1 - \frac{1}{\gamma}\omega_1 - \left(\eta - \frac{1}{\gamma}\right)\omega_1\chi_M\right)\tilde{\kappa}^M}{1 - \omega_1\left(\eta - \frac{1}{\gamma}\right)(1 - \kappa^A)} \equiv a - b\chi_M
\tag{161}
$$

where:

$$a = \frac{-\kappa_A + \left(1 - \frac{1}{\gamma}\omega_1\right)\tilde{\kappa}^M}{1 - \omega_1\left(\eta - \frac{1}{\gamma}\right)(1 - \kappa^A)}$$
$$b = \frac{\omega_1\left(\eta - \frac{1}{\gamma}\right)\tilde{\kappa}^M}{1 - \omega_1\left(\eta - \frac{1}{\gamma}\right)(1 - \kappa^A)} \tag{162}$$

Substituting this into the equation for χ_M, we obtain that:

$$\chi_M = \frac{\kappa^M + (1 - \kappa^M)\frac{1}{\gamma}\omega_1 + \tilde{\kappa}^A\left(-1 - \left(\eta - \frac{1}{\gamma}\omega_1 a\right)\right)}{1 - \omega_1\left(\eta - \frac{1}{\gamma}\right)((1 - \kappa^M) - b\tilde{\kappa}^A)} \tag{163}$$

Completing the solution. Using Proposition 1, which establishes that $\epsilon_t^S = \gamma\frac{\chi_{M,t}}{1-\chi_{M,t}}$, we obtain the result. □

D Additional Quantitative and Empirical Analysis

This Appendix provides additional details for the analysis in Section 5.

D.1 Methods and Estimation

Data. We use quarterly-frequency data from the United States from 1960Q1 to 2024Q4. We measure real GDP and the price level using data from the US BEA. From these variables, we construct GDP growth $\Delta \log Y_t$ and inflation $\Delta \log P_t$ in log differences. We measure TFP growth using the dataset of Fernald (2025), based on the work of Fernald (2014). Specifically, we take raw data on the annualized growth rate in capacity-utilization adjusted TFP and divide by 400 to obtain a comparable quarter-to-quarter growth rate $\Delta \log A_t$. Finally, as described in the main text, we construct a variable corresponding to aggregate marginal cost growth as

$$\Delta \log \mathcal{M}_t = \gamma \cdot \Delta \log Y_t - \Delta \log A_t \tag{164}$$

where we calibrate $\gamma = 0.11$ based on the findings of Gagliardone et al. (2023), who use micro-data from Belgian manufacturers to calculate the implied pass-through from the output gap to real marginal costs. This calibration is also consistent with evidence of substantial wage rigidity over the business cycle in the United States (Grigsby et al., 2021), and comparable to what one would estimate by directly looking at the relationship between detrended real wages and output in the US.[21]

Time-Varying Volatility from a GARCH Model. We estimate time-varying uncertainties regarding inflation, real output, and real marginal costs using a multivariate GARCH model. In particular, letting X_t denote the vector $(\Delta \log P_t, \Delta \log Y_t, \Delta \log \mathcal{M}_t)$, we model

$$X_t = A + BX_{t-1} + \varepsilon_t, \qquad \varepsilon_t \sim N(0, \Sigma_t), \qquad \Sigma_t = D_t^{\frac{1}{2}} C D_t^{\frac{1}{2}} \tag{165}$$

where A is a 3×1 vector of constants, B is a 3×3 matrix of AR(1) coefficients, D_t is a diagonal matrix of time-varying variances (and $D_t^{\frac{1}{2}}$ is a diagonal matrix of standard deviations), and C is a static matrix of correlations. We assume that each diagonal element of D_t, denoted as $\sigma_{i,t}^2$, evolves according to:

$$\sigma_{i,t}^2 = s_i + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i \sigma_{i,t-1}^2 \tag{166}$$

with unknown constant s_i and coefficients (α_i, β_i). Formally, this is a GARCH (1,1) model with constant conditional correlations (Bollerslev, 1990). We estimate all of the parameters

[21] For example, using this latter method, Flynn and Sastry (2022) calibrate $\gamma = 0.095$.

Table A1: Testing the GARCH Model Against Alternatives

	(1)	(2)	(3)
Model	VAR	GARCH (CCC)	GARCH (VCC)
Likelihood ratio	—	194.82	0.10
Degrees of freedom	—	6	1
p-value (χ^2(df))	—	0.000	0.746

Notes: This table presents specification tests of the GARCH model used for analysis. The data are quarterly-frequency GDP growth, GDP deflator inflation, and real marginal cost growth in the US from 1960Q1 to 2024Q4. The models are, respectively, a vector auto-regression in first differences (column 1); the same model plus a residual GARCH (1,1) with constant conditional correlations (column 2; see also Equations 165 and 166); and the same model plus varying conditional correlations (column 3). The second row gives the likelihood ratio for the model in question versus the nested model in the previous column. The third row gives the degrees of freedom of the likelihood ratio test, equal to the number of additional free parameters. The fourth row gives the p-value from evaluating the test statistic at the χ^2 distribution with the corresponding degrees of freedom.

via joint maximum likelihood.

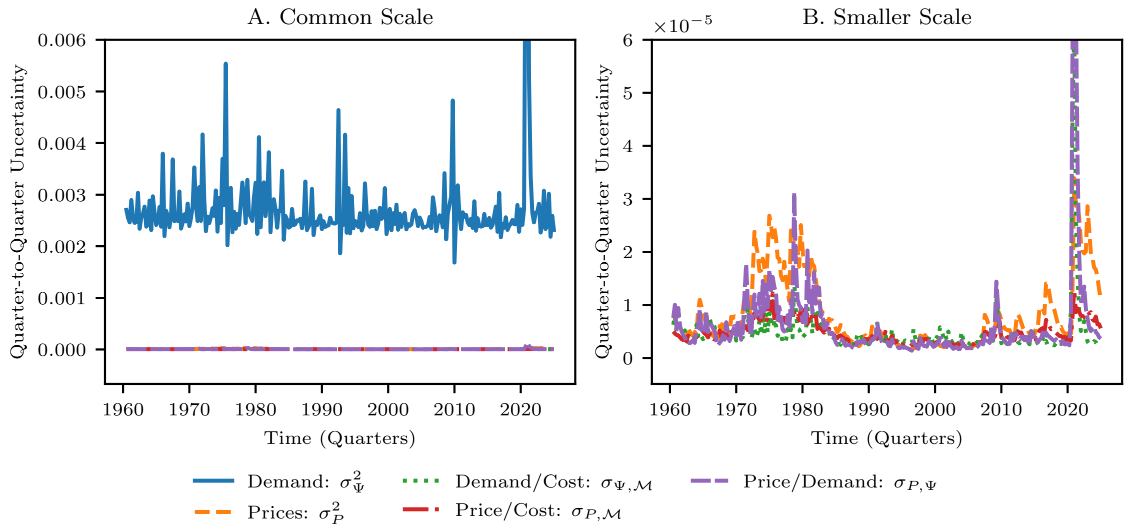
In calibrating the model, we use volatilities dated at time t to stand in for economic agents' uncertainty about making decisions at time t. As is apparent from Equation 166, these volatilities are measurable in macroeconomic history up to period $t-1$. Thus, this timing convention is consistent with our notion in the model that economic agents observe all macroeconomic history up to time $t-1$ and their priors are informed by these observations. All in all, for each quarter t, we set

$$\begin{aligned} \hat{\sigma}^2_{\Psi,t} &= \hat{\Sigma}_{Y,Y,t} + R^2\hat{\Sigma}_{A,A,t} \qquad & \hat{\sigma}_{\Psi,P,t} &= \hat{\Sigma}_{Y,P,t} \\ \hat{\sigma}_{\mathcal{M},\Psi,t} &= \hat{\Sigma}_{\mathcal{M},\mathcal{M},t} & \hat{\sigma}_{\mathcal{M},P,t} &= \hat{\Sigma}_{\mathcal{M},P,t} \end{aligned} \tag{167}$$

where the $\hat{\Sigma}_{\cdot,\cdot,t}$ are the elements of the residual covariance matrix and $R = 6.5$ from the quantitative estimates of Bloom et al. (2018).

Our estimation procedure allows us to naturally test the specified model against nested alternatives (A1). In column 2, we compare our GARCH model with the nested model with constant volatility: a vector auto-regression (VAR) in first differences for the variable X_t. This model has six fewer parameters, corresponding to the ARCH and GARCH parameter in each residual's equation. The likelihood ratio of 194.82 comfortably rejects the nested VAR model. In column 3, we compare the constant conditional correlations GARCH model (our baseline) with an expanded model that allows for varying conditional correlations (Tse and Tsui, 2002). In particular, in this model, the covariance matrix of residuals is now

Figure A1: Estimates of Time-Varying Uncertainty



Notes: Both panels plot our quarterly time-series estimates of uncertainty, estimated as described in this appendix. All lines are computed from one-quarter-ahead volatility predictions from a constant conditional correlations (CCC) GARCH model. The left plot shows all series on a common scale, and the right plot zooms in on the series other than demand. Both plots feature spikes that are off the scale of the graph during the Covid-19 lockdown.

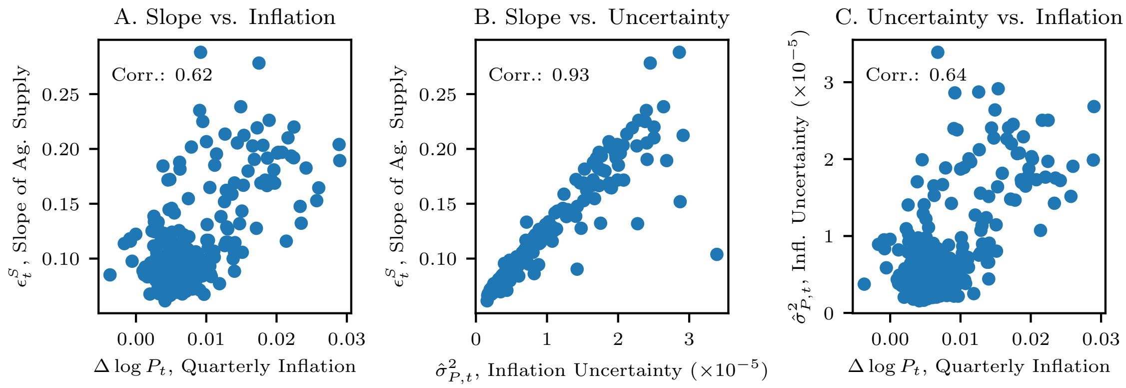
$\Sigma_t = D_t^{\frac{1}{2}} C_t D_t^{\frac{1}{2}}$ (cf. Equation 165) where

$$C_t = (1 - \lambda_1 - \lambda_2)C + \lambda_1 \Psi_t + \lambda_2 C_{t-1} \tag{168}$$

where $\lambda_1, \lambda_2 \geq 0$ are parameters governing the dynamics of the correlations, which satisfy the restriction $0 \leq \lambda_1 + \lambda_2 < 1$; C is a long-run mean of the correlations; and Ψ_t is a 4-period (number of variables plus one) rolling estimator of the standardized residuals $\tilde{\varepsilon}_t = D_t^{-\frac{1}{2}} \varepsilon_t$. Due to the additional restriction on λ_1 and λ_2, this model has only one more free parameter than the nested constant conditional correlations model. The likelihood ratio of 0.10 demonstrates an only marginal improvement in fit, failing to reject at conventional significant levels. Thus, the data suggest that a model with time-varying volatility, but constant conditional correlations, is a good fit for recent US history.

Estimates of Time-Varying Uncertainty. In Figure A1, we plot the raw time series for each of our uncertainty measures. We observe that our estimates of demand uncertainty are an order of magnitude larger than our estimates of other uncertainties. This is natural given our large assumed value of R, the (square root of the) ratio between idiosyncratic demand uncertainty and aggregate real marginal cost uncertainty. But this does not necessarily imply

Figure A2: Estimates of Time-Varying Uncertainty

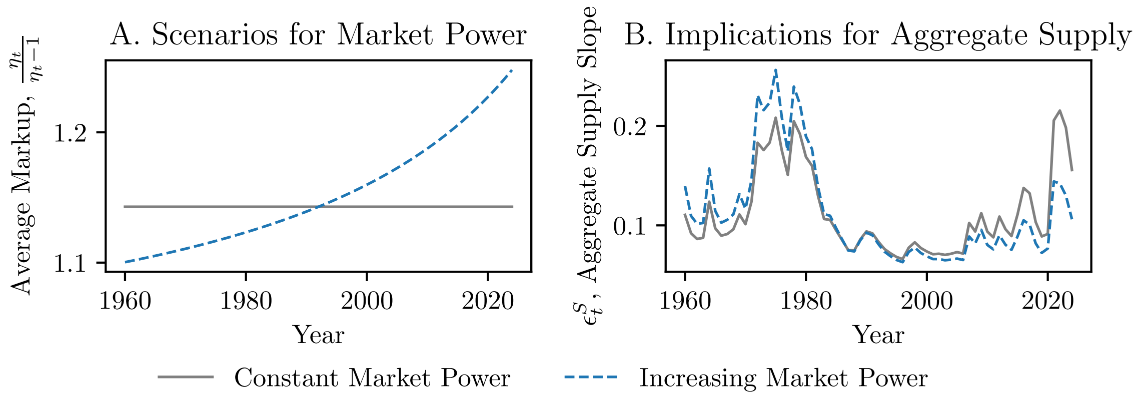


Notes: This figure shows the bivariate relationships between the estimated slope of aggregate supply (see Section 5.1 and Appendix D.1), the level of inflation (quarterly log difference in GDP deflator), and uncertainty about inflation (estimated one-quarter-ahead from a constant conditional correlations GARCH model; see Section 5.1 and Appendix D.1). Each observation corresponds to one quarter. The numbers in the top left indicate the correlations for each pair of variables.

that demand uncertainty is the only influential force shaping the slope of microeconomic or macroeconomic supply, since uncertainties enter our formulas in interaction with the elasticity of demand η. This is apparent from our results—the fluctuations in the slope of aggregate supply in Figure 4 clearly reflect significant fluctuations in the other components of uncertainty that are plotted in the second panel of Figure A1.

Inflation Levels, Inflation Uncertainty, and the Estimated Slope. Figure A2 shows the correlations between our estimated slope, the level of inflation, and uncertainty regarding inflation. Broadly speaking, we estimate the slope of aggregate supply to be high when the level and uncertainty regarding inflation are high (panels A and B). Moreover, the level of inflation and inflation uncertainty are highly correlated with one another (panel C). This finding echoes the observation of Ball et al. (1988) that it is difficult, empirically, to find circumstances in which levels and volatilities of inflation are decoupled from one another, posing a difficulty for testing different models of state-dependent aggregate supply against one another. However, as observed in Section 5.3, our model based on *relative* uncertainty gives quite different predictions than simple models based on the level of or one-dimensional uncertainty regarding inflation when confronted with global data.

Figure A3: Rising Market Power and Flattening Aggregate Supply



Notes: This figure plots the inverse elasticity of aggregate supply under different scenarios of declining market power and shows how trends in market power affect the slope of aggregate supply. We calibrate the model under two scenarios: a fixed value of $\eta_t \equiv 8$ (grey line, "Constant Market Power") and a linear trend over the sample from an initial value of $\eta_{1960Q1} = 11$ to a final value of $\eta_{2024Q4} = 5$ (blue dashed line, "Rising Market Power"). All other parameters, including the measured uncertainties, are exactly as in our baseline calculations (see Section 5.1 and Appendix D.1). Panel A shows the time series behavior of average markups, $\frac{\eta_t}{\eta_t - 1}$, implied by our different assumptions about the elasticity of demand. Panel B shows the resulting calculations for the slope of aggregate supply, averaged over years.

D.2 Market Power and Aggregate Supply

A recent literature has suggested that market power, as measured by rising markups, has risen throughout time (De Loecker et al., 2020; Demirer, 2020; Edmond et al., 2023). Combined with our theoretical finding that increased market power flattens aggregate supply under plausible parameter values, this suggests another potentially relevant culprit for the long-run flattening of supply.

To study this possibility, we consider alternative calibrations of the slope of aggregate supply in which we allow a secular downward trend in the elasticity of demand. Specifically, we consider a scenario in which η linearly declines from 11 to 5 between 1960 and 2024. This implies an increase in average markups from $11/10 = 1.10$ to $5/4 = 1.25$. These exercises are *not* counterfactuals, which would require fully estimating the model and accounting for the effects of market power on macroeconomic uncertainty. Instead, they are alternative calibrations that would be more appropriate than our baseline if the elasticity of demand has truly fallen over time.

Introducing a decline in market power increases the slope of aggregate supply in the 1970s and decreases the slope in modern times (Figure A3). Calibrating to this different scenario implies that the slope of aggregate supply flattens by 41% from 1978-1990 to 1991-2018, compared to an estimate of 28% in our baseline model and an empirical estimate of 51%

from Hazell et al. (2022). Thus, allowing for an increase in market power allows the model to more closely match empirical estimates for the flattening of aggregate supply from the 1970s to the 2010s. These calculations provide suggestive evidence that market power interacts in a quantitatively relevant way with the slope of aggregate supply in our model. We leave further analysis of this interaction to future work.

D.3 International Evidence

Data. We take annual data from 1960-2019 from the most recent edition of the Penn World Tables (Feenstra et al., 2015; Zeileis, 2023). In particular, we measure real GDP, GDP deflator (expressed in local currency), total hours, and the real value of the capital stock. We construct real GDP growth and inflation as log differences (annual) in the corresponding variables. We calculate TFP at the level of countries c and years t based on a constant labor share of 2/3 as

$$\log A_{ct} = \log \text{RealGDP}_t - \frac{1}{3} \log \text{RealCapitalStock}_{ct} - \frac{2}{3} \log \text{LaborHours}_{ct} \tag{169}$$

Finally, we construct growth in real marginal costs as described in Equation 164, using the same calibration for γ. To calculate the slope of aggregate supply in each country, we also carry over our calibration of $\eta = 8$, $R = 6.5$, and $\kappa^M = 0.29$.

Volatility from a VAR Model. Because our interest is cross-sectional differences, we estimate a VAR model with time-*in*variant volatility for each country, rather than a model of time-varying volatility (e.g., a GARCH model). In particular, letting X_t again denote the vector $(\Delta \log P_t, \Delta \log Y_t, \Delta \log \mathcal{M}_t)$, we model

$$X_{ct} = A_c + B_c X_{c,t-1} + \varepsilon_{ct}, \qquad \varepsilon_{ct} \sim N(0, \Sigma_c), \tag{170}$$

where (A_c, B_c) are country-specific coefficients and Σ_c is a country-specific covariance matrix. We map the covariances from the VAR to the model using the same method described in Equation 167, but with an estimate for Σ that depends on countries rather than time periods.

Finally, we drop three outliers from our calculations, Greece, Iceland, and Sweden, for which we calculate a slope of aggregate supply and/or inflation-output relationship more than 3 standard deviations away from the median.

Empirical Proxies for the Slope of Aggregate Supply. We calculate two country-level proxies for the slope of aggregate supply. The first is the country-level, reduced-form relationship between inflation and real output growth. That is, the coefficient β_c^S from the regression

$$\Delta \log P_{ct} = \alpha_c + \beta_c^S \cdot \Delta \log Y_{ct} + \varepsilon_{ct} \tag{171}$$

estimated by ordinary least squares for each country c, using variation across time periods. The coefficient

$$\beta_c^S = \frac{\text{Cov}[\Delta \log P_{ct}, \Delta \log Y_{ct}]}{\text{Var}[\Delta \log Y_{ct}]} \tag{172}$$

measures the strength of the reduced-form relationship between real output growth and inflation. This is in the spirit of the reduced-form tests of Lucas (1973) and Ball et al. (1988), who similarly look at covariances of real and nominal components of GDP. To understand the structural interpretation of β_c^S, we observe from Theorem 2 that, in the equilibrium of the model,

$$\Delta \log P_t = \epsilon^S \Delta \log Y_t + \underbrace{\left(\delta_t \Delta \log A_t + \Delta \log \bar{P}_t\right)}_{=\tilde{\varepsilon}_{ct}} \tag{173}$$

where the term in parenthesis can be interpreted as the structural residual of Equation 171. Intuitively, the structural residual of the reduced-form relationship between aggregate prices and aggregate quantities can be thought of as the "shock to aggregate supply," and the reduced-form relationship traces out the "aggregate supply curve" if and only if all variation in real GDP growth is induced by "aggregate demand" shocks (*i.e.*, money supply shocks). If this does not hold (*i.e.*, if some variation in real GDP growth, in deviation from the mean, is driven by productivity), then we expect $\text{Cov}[\Delta \log Y_{ct}, \tilde{\varepsilon}_{ct}] < 0$ and a downward bias in the ordinary least squares estimate, or $\text{plim}\ \hat{\beta}_c^{S,OLS} < \epsilon^S$

As a second strategy, we construct a model-based instrument for money supply growth. Using the money demand equation (the second equation in Proposition 1), we observe that, in equilibrium,

$$M_t = Y_t^{\gamma} P_t \frac{1 + i_t}{i_t} \tag{174}$$

Abstracting from nominal interest rate changes, which is what our model implies under the imposed simplification of time-invariant volatility (and time-invariant ϵ^S), the model implies

$$\Delta \log M_t = \gamma \Delta \log Y_t + \Delta \log P_t \tag{175}$$

and moreover, due to the random-walk behavior of the money supply, that these increments are idiosyncratic across time and uncorrelated with shocks to productivity. Therefore, we construct the money growth instrument $\Delta \log \tilde{M}_{ct} = \gamma \Delta \log Y_{ct} + \Delta \log P_{ct}$ and use it as an instrument for real GDP growth. The first-stage equation is

$$\Delta \log Y_{ct} = \zeta_c + \beta_c^F \cdot \Delta \log \tilde{M}_{ct} + \nu_{ct} \tag{176}$$

Table A2: Predicting the Slope of Aggregate Supply

	(1)	(2)	(3)	(4)	(5)	(6)
	Outcome is $\hat{\beta}_c^S$ (Reduced-form)			Outcome is $\hat{\beta}_c^{S,IV}$ (Structural)		
$\hat{\epsilon}_c^S$, Slope of ag. supply	0.261	0.132	0.0174	7.226	7.618	8.823
	(0.0379)	(0.0392)	(0.0565)	(2.410)	(3.406)	(4.996)
Mean inflation		-7.542			22.86	
		(1.583)			(137.7)	
Inflation uncertainty			-84.96			556.1
			(17.14)			(1516.1)
Observations	29	29	29	29	29	29
R^2	0.638	0.807	0.814	0.250	0.251	0.254

Notes: This table reports the cross-country relationship between empirical and theoretical proxies for the slope of aggregate supply. All estimates are from linear regressions where the unit of observation is an OECD country. In columns 1-3, the outcome is the "reduced-form" slope of aggregate supply defined in Equation 172. In columns 4-6, the outcome is the "structural" slope of aggregate supply defined in Equation 177. The independent variables are the model-implied slope of aggregate supply, calculated based on a macroeconomic calibration and measurements of relative uncertainty in each country; the mean value of GDP deflator inflation from 1960-2019; and the one-step-ahead forecast variance of inflation from a three-variable VAR model (see Equation 170) over the same period. Standard errors are in parentheses.

and the structural equation remains Equation 171. The population two-stage least squares coefficient of the slope of supply is

$$\beta_c^{S,IV} = \frac{\text{Cov}[\Delta \log P_{ct}, \Delta \log \tilde{M}_{ct}]}{\text{Cov}[\Delta \log Y_{ct}, \Delta \log \tilde{M}_{ct}]} = \frac{\gamma \text{Cov}[\Delta \log P_{ct}, \Delta \log Y_{ct}] + \text{Var}[\Delta \log P_{ct}]}{\text{Cov}[\Delta \log P_{ct}, \Delta \log Y_{ct}] + \gamma \text{Var}[\Delta \log Y_{ct}]} \tag{177}$$

Cross-Country Evidence. Table A2 summarizes the relationship between our empirical proxies and model-based calculations for the slope of aggregate supply. Column 1 shows the positive relationship between the reduced-form slope and model-based slope that is visualized in the left panel of Figure 5. This relationship is robust to controlling for the level of inflation (column 2). The relationship becomes statistically insignificant when controlling for one-step-ahead inflation uncertainty (column 3), although the coefficient on the latter is inconsistent with the theoretical prediction. Turning to the structural estimates (columns 4-6), we estimate a large and quantitatively stable relationship between the data-based and model-based estimates. The larger magnitudes in columns 4-6 versus 1-3 are consistent with the hypothesis that the reduced-form coefficients are biased toward zero by spurious correlation with aggregate supply shocks. The coefficients on mean inflation and inflation uncertainty in columns 5 and 6 are consistent with theory, but imprecisely estimated and of marginal consequence to the R^2 of the model.

While these results are to be interpreted with caution, given the limited sample size and abundance of confounding factors in cross-country analysis, they offer suggestive evidence that the model-based slope of aggregate supply helps predict cross-country variation in the inflation-output relationship. Moreover, our model's prediction based on relative variance has predictive power over and above the mean and one-step-ahead uncertainty regarding inflation, which are the main factors influencing the slope of aggregate supply in other theories of state-dependent firm adjustment (Ball et al., 1988). Further investigation of the differences between these models may be possible by incorporating both time-series and cross-sectional variation in an international panel or by turning to micro data. We leave these investigations to future work.

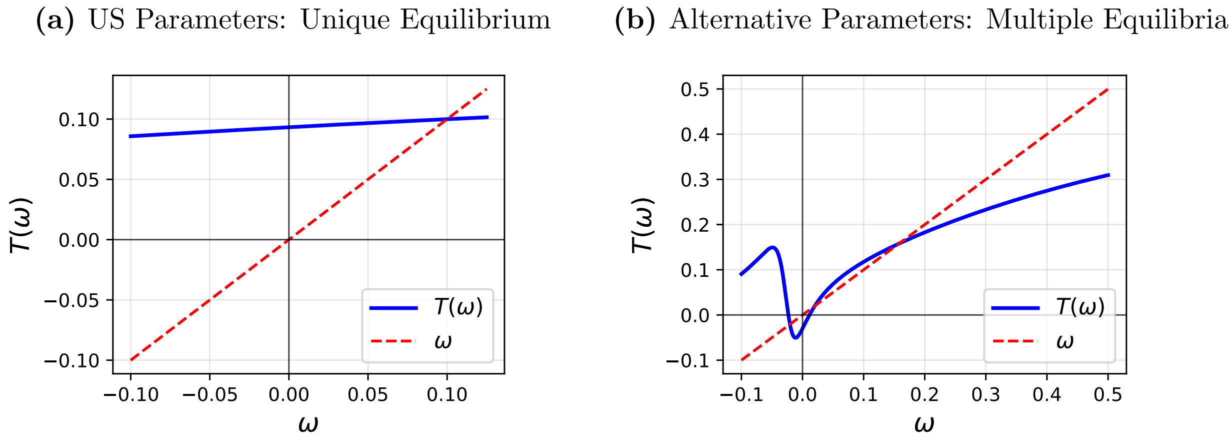
D.4 Counterfactual Analyses and Equilibrium Multiplicity

The analysis in the main text leveraged the "reduced form" uncertainties relevant to the firm in Theorem 1 and did not estimate the structural uncertainties that mediate firms' supply functions slopes via the fixed point in Theorem 3. As noted in the main text, an advantage of this approach is that the economic analyst can measure the slope of firms' supply functions without taking a stance on the general equilibrium features of the economy. Moreover, by using such observational data, the analyst can bypass issues of equilibrium selection.

However, a limitation of this approach is that this method precludes conducting counterfactuals which would be relevant when model parameters endogenously respond to policy. In this section, we outline how one can use our theory to conduct counterfactual exercises and demonstrate that a unique equilibrium exists for a reasonable calibration of the US economy.

Methodology and Calibration. Solving the fixed point in Theorem 3 requires values for the preference parameters (η, γ) and uncertainties $(\sigma_{\vartheta,t}, \sigma_t^A, \sigma_t^M, \kappa_t^A, \kappa_t^M)$. To simplify the analysis, we assume that these parameters are time-invariant. We set $\eta = 8$ and $\gamma = 0.11$ as in the main text. Moreover, we set $\sigma_{\vartheta,t}^2 = 0.0026$ to match the unconditional mean of our GARCH estimates in Section 5 over our sample period. Next, we back out the latent aggregate demand shock using the observation that $M_t = \frac{1+i_t}{i_t} C_t^\gamma P_t$ from Proposition 1. We calibrate σ_t^M to match the mean unconditional variance in $\Delta \log M_t$ following Equation 16. We also calibrate σ_t^A to match the mean unconditional variance in $\Delta \log A_t$, where we measure TFP growth A_t using the dataset of Fernald (2025). Finally, we set κ_t^M and κ_t^A to zero. We do so to keep our analysis consistent with the methodology of Golosov and Lucas (2007), which directly estimates firms' uncertainty using realized inflation rates. Nevertheless, the basic message of equilibrium uniqueness is not sensitive to these parameter choices.

Figure A4: Fixed Point of Firms' Supply Function Slopes



Notes: The left panel plots the fixed point from Theorem 3 under a parameterization with a unique equilibrium when parameters are calibrated to the US economy: $(\kappa^A, \kappa^M, \sigma_M^2, \sigma_A^2, \eta, \gamma, \sigma_\vartheta^2) = (0, 0, 0.00017, 0.000068, 8, 0.11, 0.0026)$. The right panel plots a parameterization with three equilibria: $(\kappa^A, \kappa^M, \sigma_M^2, \sigma_A^2, \eta, \gamma, \sigma_\vartheta^2) = (0.1, 0.9, 5, 10, 2, 0.02, 5)$.

Results. Figure A4 plots the fixed point in Theorem 3 for various parameter values. The left panel depicts the fixed point for a parameterization of the US economy, described above. The right panel depicts the fixed for an alternative parameterization which features multiple equilibria.

Observe that the US parameterization features a unique equilibrium. The intuition for this result is that idiosyncratic demand uncertainty is large in our estimation relative to other sources of uncertainty. For this reason, the fixed point for firms' microeconomic supply elasticities is well approximated by a linear function. To obtain equilibrium multiplicity, we have had to increase firms' relative uncertainty about aggregate *vs.* idiosyncratic demand conditions by more than ten-fold, as well as fix a particularly low value of γ. To provide some intuition for multiplicity in this environment, observe that the dynamics of real GDP are described by the following Equation (see Lemma 1):

$$\log C_t = \tilde{\chi}_{0,t} + \frac{1}{\gamma} \frac{\kappa_t^A}{1 - \omega_{1,t}\left(\eta - \frac{1}{\gamma}\right)(1 - \kappa_t^A)} \log A_t + \frac{1}{\gamma} \frac{(1 - \kappa_t^M)(1 - \eta\omega_{1,t})}{1 - \omega_{1,t}\left(\eta - \frac{1}{\gamma}\right)(1 - \kappa_t^M)} \log M_t \quad (178)$$

In particular, higher values of $\omega_{1,t}$ reduce the volatility of aggregate consumption that arises through productivity shocks. But this in itself is a force that favors steeper supply functions, since even small shifts in demand are likely to imply large changes in marginal costs. Consequently, this economy can feature multiple equilibria in firms' supply function elasticities, through a general equilibrium feedback loop that arises between supply function choice and firms' endogenous uncertainties, as we described in Section 4.4. Nevertheless, we have found it challenging to construct examples with multiple equilibria and quantitatively reasonable parameter values for the US. Consequently, we believe that our framework is also amenable to counterfactual analyses for the US economy.

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